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A HISTORY OF MATHEMATICS

First Part — Arithmetic (Pāṭīgaṇita)

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by

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Complete English Translation

Rendered from the 1910 Benares edition
Ganita kā Itihāsa, Pahlā Bhāg, Pāṭīgaṇita

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Translator’s Note

This volume is an English rendering of Mahamahopadhyaya Sudhakara Dvivedi's Hindi book *Ganita ka Itihasa*, Part I (*Patiganita*), printed at Benares in 1910. The 1910 book is 207 pages of large Hindi type with plates and a word-index. This English edition translates the whole treatise from the opening anka chapter through the closing lives; it does not reprint the Hindi index or redraw every worn woodcut. Dvivedi (1855-1910) was professor of mathematics and astronomy at the Government Sanskrit College, Benares, editor of major jyotisa texts, and author of the biographical *Ganaka-tarangini*. He dated the preface 29 October 1910 and died the same year; the algebra, geometry and trigonometry parts he announced were never published.

The original is a world history of arithmetic written for Hindi-reading paṇḍits and examination candidates, with special weight on Indian sources. Dvivedī compares Babylonian, Egyptian, Greek, Roman, Chinese, Tibetan, Arabic, Russian and Indian numerals; traces zero and place-value; records Sanskrit names of high decimal places; and treats the operations, roots, fractions, proportion, series, interest, magic squares, combinations and logarithms, closing with brief lives of authors named in the book.

The translation follows the Hindi argument closely: order of topics, quotations, dates, and Dvivedī’s own judgements are kept. Verse is given in English sense-translation, with the first words of well-known ślokas noted where useful. Tables of foreign glyphs cannot always be redrawn from the damaged 1910 blocks; they are described in words and, where Dvivedī also printed Roman or Arabic forms, those forms are used. Occasional 1910 misprints and European names in old Hindi spelling are silently regularized (Aristotle, Ptolemy, Gerbert, Lucas de Burgo, and so on) when the intended person is clear.

Pāṭīgaṇita is the traditional Indian name for arithmetic done on a board (*pāṭī*). Dvivedī notes that the same subject is also called *vyakta-gaṇita* (manifest mathematics) and, in some places, *aṅka-gaṇita*.

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Preface

Bhārata maĩ naranāri muhĩ bāta bāta mẽ Rāma
jo rājata ghara ghara sadā tāhi karaũ paranāma

Every year, for the candidates sitting the Jyotiṣācārya examination of the Government Sanskrit College at Benares, I give some lectures on the history of mathematics. Last year, while lecturing, the wish arose that I should write a book in the Hindi language, so that some service might be done to my own countrymen. For that reason I have today had this first part, on Pāṭīgaṇita, printed and set before the learned.

There is here no Asiatic Society in whose library the books of the paṇḍits might be found. For that reason, in making the signs and figures I have had to lift a great burden; if my opinion should prove wrong, let it be corrected at once.

In this book I have written those matters which came to my mind, and also those which I found after reading old Sanskrit texts and the standard European books. The names of the European paṇḍits which I had seen only in English letters I have given with as exact a Hindi spelling as I could, so that a Hindi reader who hears the European name may recognize it. I have also given, for those European names whose Hindi form has become current, the original English spelling in brackets, so that a reader of English letters may know the true name.

Where possible I have added at the end short lives of the paṇḍits named, and an alphabetical index of the special words of the book, so that scholars need not labour to find a word.

At the close I request the paṇḍits: wherever some error has crept in, correct it; and gird yourselves to write on this subject a book better than this one. I have only laid a foundation in this country. You may raise a palace upon it, or you may dig the foundation up and wash it away.

It is my wish afterwards to write the second part (algebra). The first part will be Pāṭīgaṇita, the second Bījagaṇita, the third Rekhāgaṇita and Kṣetra-vyavahāra, and the fourth Trikoṇamiti and Jyautiṣa-siddhānta — these four parts will make up this History of Mathematics.

Jo ḍhūḍhā so pāiyā˜gahire pānī paiṭhi
ve bapurī kyā pāiyā˜rahī˜kināre baiṭhi
— Kabīr

29-10-1910 Sudhākara Dvivedī

A History of Mathematics

First Part — Pāṭīgaṇita

Jayati jagati citraṃ yac caritraṃ pavitraṃ
sura-sad-janam unnītaṃ saj-janānanda-nītam |
tam iṣṭaṃ hṛdi nitāntaṃ satya-satīśa-mukāntaṃ
likhati gaṇita-vṛttaṃ sthāpayitvā mudāsam ||

Pāṭīgaṇita is also called vyakta-gaṇita, and in some places aṅka-gaṇita.

Close translation of the 1910 text, printed pages 1 to 30

These pages are the opening of the 1910 Benares edition after the preface: why the ten signs exist; the puranic and ritual pictures of the digits; the Sanskrit etymologies of eka to nava; Sriharsa's sloka on the ear and the nine; how letters were cut on palm-leaf; then the numerals of Babylon, Egypt, Greece, the Sabaeans, Rome, China, Tibet and the Basques; the Hindu, Arabic and Russian signs; and the Brahmi and Kharosthi of Asoka's pillars, down to the old barakhadi drill of Benares.

The digits (anka) (pp. 1-5)

When, for the business of this world, words were made, if you think with attention you will see that before that the words for one, two, three and the rest must first have been made for the understanding of those numbers. The moment a child comes into the womb the counting of months — one, two — begins.

Many people say that first there were nada and bindu, and afterwards the four Vedas were made. However that may be, with nada, at the very sounding of the seven notes, the digit seven comes; and with the Veda, in saying 'the four Vedas,' the digit four comes. In saying Vedatrayi, three has already come.

When the fourteen sutras of grammar issued from the dhakka of Mahadeva, in Sanskrit caturdasa was made. In the naming of Panini's grammar, in saying Astadhyayi, the word for eight was made; in the singular and the dual, one and two came. The Nirukta is itself a Vedic lexicon; in it 'imani prthivi-namadheyany ekavimsatih,' 'hiranya-namany uttarani panca-dasa,' 'kantikarmanah uttare dhatavo 'stadasa,' 'gati-karmane uttare dhatavo dvavimsati-satam' — in these the digits themselves are filled. In Siksa, 'trisastir va catuhsastir varnah sambhu-mate matah' — in this sixty-three and sixty-four came. In the laksana of the chief metre, Gayatri, 'iha hi sad-aksaro gayatri-caranah' — in this six came. In Kalpa, in the making of the vedis, wherever you look, the digit itself is principal.

In Sankhya, in saying 'panca bhutani' and 'dasendriyani,' five and ten came. In Yoga, in saying 'sad cakrani,' 'asta kamalani,' 'dasa randhrani,' six, eight and ten come at once. In Purva-Mimamsa, because karma is principal, everywhere there are digits and only digits. In Vedanta (Uttara-Mimamsa), in saying advitiya, one and two become chief. In Nyaya and Vaisesika, in saying twenty-four gunas, dvy-anuka, try-anuka, twenty-four, two, three come. In saying 'eighteen Puranas,' eighteen comes.

In Smrti, that is Dharmasastra, in the description of triratra, paksini, dasaha, yuga, wherever you look only digits are seen. In the Ramayana, Visvamitra's word to Dasaratha, 'iyam tv aksauhini purna balasya mama durjaya'; the fourteen years of Rama's forest-dwelling; after Dasaratha's death the dvadasaha asauca; and in the Kiskindha-kanda the counting of the vanaras — all are digits. In kavya and nataka, in the counting of sargas and ankas, there are digits and only digits. In the Mahabharata the five Pandavas, the aksauhini, twelve years of forest-dwelling, the Visnu-sahasra-nama — in these all are filled with digits. In Vaidya-sastra, in making rasas, wherever you look there are digits. In the vidya of weapons, in putting water on iron, in heating in the fire, there are digits themselves.

From the things written above it is certain that in the world no behaviour can happen without counting. The charpai on which you sleep and the house in which you live also cannot be made without the digit. The desire of knowing how those digits were made, by which all the behaviours of the world are bound — I understand that this desire will be in small and great alike.

First of all this question rises: in doing the counting, why were only these ten signs made — nine digits and one sunya? In the answer this thing comes into the mind: a man, as far as he can, does not wish the help of another. For that work which can be done easily by himself, what need of another? Therefore people first of all, for counting, brought into use the fingers of both their hands; and after knowing the ten, afterwards, for making others understand, in the place of these nine fingers the marks of one, two and the rest were made; those same in Sanskrit they call anka. This word anka is made from the dhatu anki, whose meaning is 'to

make a mark.'

yatha sikha mayuranam naganam manayo yatha |
tad-vad vedanga-sastranaṃ ganitam murdhani sthitam ||

(Therefore it is written in the Jyotisa-vedanga: as the crest of peacocks, as the jewels of nagas, so among the sastras of the Vedanga, mathematics stands on the head.)

Mundhe mundhe matir bhinna — that is, the intellect of every man is separate; therefore the method of making nine digits was not born the same in every place. In some places, counting by the fingers of one hand only, only five digits were made. Somewhere, after knowing the nine digits and the ten, for their own ease, digits of eleven-fold, twelve-fold, sixty-fold were made.

Aristotle also, who lived 340 years before the Christian era, raised this same question in his book named Problemata: why among all men there are signs of counting only up to ten. In the answer Aristotle also wrote this same thing: all people took the fingers of their own hands in doing the counting, therefore the ten signs were made; but somewhere, because of boys not remembering in that way, the digits of counting were made a little fewer — as in the Thracian people there are digits only up to four. From this it is clear that from long before Aristotle the ten was current.

In their own countries the intelligent made separate marks of these digits. In the writings found on pillars or stones of the time of King Asoka, marks of this kind and of that kind are found. In this way, before the vidya of writing letters, in separate countries separate signs of digits were made. Afterwards, when letters were made for the writing of words, somewhere the old signs remained, and in other places they began to publish the digits by the shape of the first letter of their words; then, in writing quickly quickly, the shape of the first letter going on spoiling, they became the 1, 2, 3, 4, 5, 6, 7, 8, 9 of today.

Before proving the matter above, some digit-signs of our own and of other countries are shown.

Puranic and ritual forms (p. 5)

The opinion of the Pauranikas is that the spoiled shape of those pictures of the nine grahas which are written in the Puranas for worship is 1, 2, 3. The spoiled shape of the pictures of the grahas which the karmakandis make in homa and the rest is 1, 2 — this thing appears impossible.

Etymologies of the Sanskrit number-words (pp. 6-9)

Eka — from the dhatu i (to go), with the Unadi ka; that which goes is eka. Some take it from anj (to make clear).

Dvi — according to Panini, from the dhatu du (gatau) with the pratyaya i (ac i), whose meaning is 'that which goes.' According to Hemacandra, from the dhatu us (purane) with the pratyayas ha and jna, dvi and tri are made (used-dvitros ca), whose meaning is 'that which fills.' In the famous Sabda-cintamani kosa of Gujarat, from the dhatu had (vidarane adare ca) with the pratyaya di, dvi is made, whose meaning is 'that which splits' or 'that which honours.'

Tri — from the dhatu tr (plavana-taranyoh) with the pratyaya di. In the Unadi there is for this the very sutra tarater dih. That which swims on water is tri.

Catur — from the dhatu cat (yacane) with the Unadi pratyaya tur. That which begs on all four sides is catur.

Panca — from the dhatu paci (vistare) with the Unadi pratyaya an. That which is spread is panca.

Sat — from the dhatu ghat (avayave) with the Unadi pratyaya ksip. That in which there are avayavas (several parts) is sat.

Sapta — from the dhatu sap (samavaye) with the Unadi pratyaya kanin and the bringing of tuk. Bhattoji Diksita writes in his Siddhanta-kaumudi: samavayah sambandhah samyag avabodho va — that is, from samavaya a relation or a knowing well is taken. Therefore that by which knowing well is done is sapta.

Asta — from the dhatu asu (vyaptau samghate ca) with the Unadi pratyaya kanin and the bringing of tuk. That which is present (vyapta) in every place is asta.

Nava — from the dhatu nu (stutau) with the Unadi pratyaya kanin. That which is worthy of praise is nava.

In the time of Sriharsa our people's ear was of the same shape as the written 'nava.' In the 61st sloka of the seventh sarga of his kavya the Naisadha-carita he gave the upama of Damayanti's ear with the digit nava:

asyas yad astadasa samvibhajya vidyah sruti dasa-tur-ardham ardham |
karnantar utkirna-gabhira-rekhah kim tasya samkhyeva na va navankah ||

When they did not know how to make paper, at that time on the leaf of the tala they used to cut the shapes of letters by digging with a pointed iron pen. So that those shapes should be seen well by the eyes, they used to smear the cut tala-leaves with haritala (orpiment).

The numerals of the Babylonian astronomers (pp. 11-14)

In Babylonia the numbers found to this day are all below ten lakh. Two tables of that place have also been found. In the first the squares from one to sixty are written. It is possible that this table was made between 2300 and 1600 years before the Christian era. In this table the places of the digits are sixty-fold; because in it 8^2 (=64) is written 1, 4; in the same way 9^2 = 1, 21; 10^2 = 1, 40; 11^2 = 2, 1 — written in this way.

Among the Hindus also, in the mathematics of the planets and in the time of ista, the method of sixty-fold places of digits is current to this day. In Sanskrit they write 64 kala as 1;4. They write the digit of the greatest place highest of all, and that less than it below. Among the Hindus the sixtieth part of a day is called ghati, the sixtieth part of a ghati pala, and the sixtieth part of a pala vipala. In the Kalapada of the Aryabhatiya it is written:

sastir nadyo divasah sastis tu vinadika nadi |

In the current Surya-siddhanta it is written that the knowledge of the mathematics of the planets Maya received from the Sun; then Maya spread that mathematics in the midnight of Lanka (see my commentary Sudhavarsini on the Surya-siddhanta). From finding the trace it is known that the time was Greek. In Sanskrit people used to call the Greek Yavana or Mleccha. Varahamihira wrote in his Brhatsamhita:

mleccha hi yavana tesu samyak sastram idam sthitam |
rsivat te 'pi pujyante kim punar daivavid dvijah ||

Many hold that the Greek pandits Hypsicles and Ptolemy brought this sixty-division (kala, vikala) from Babylonia to their own country; then from there that same method spread in Hindustan also. However that may be, that 360 saura days make one saura year is written in the Rgveda also:

dvadasa pradhayas cakram ekam trini nabhyani ka u tac ciketa |
tasmin sakam trisata na sankavo 'rpitah sastir na calacalasah ||
(Rgveda 1.164.48)

In the second table of Babylonia the white measures of the moon's orb from the end of amavasya to the end of purnima are written. From the pratipada of sukla-paksa to pancami the white measures 5, 10, 20, 30, 1;20 (=80) are written. Then further, in every tithi, the white measures are written with an increase of sixteen and sixteen. In this table the moon's orb is written 240. Until the first five tithis the white is written in a geometric series, afterwards in an additive series.

Egyptian numerals and words (pp. 14-17)

The people of Egypt wrote a stroke for one, a heel-bone for ten, a coil of rope for a hundred, a lotus for a thousand, a bent finger for ten thousand, a tadpole for a lakh, and a figure of a seated man for ten lakh. Their number-words were built in the same ten-fold way. The book of Ahmes (the Rhind papyrus) shows unit fractions and the heap (hau) as the unknown; that book is treated later with the fractions. From it this much is certain here: the people of Egypt knew a written mathematics of digits several thousand years before the Christian era, but they did not have the nine digits and the sunya of the Hindus.

The numerals of the land of Greece (pp. 17-21)

The people of Greece first used strokes and then the letters of their alphabet for numbers: alpha 1, beta 2, and so on, with a mark to show that the letter is a number. They also had acrophonic signs from the first letter of the number-word. They had no separate sign for sunya. Their intercourse with Hindustan is very old. In the Mahabhasya on Panini's Astadhyayi it is written:

> *savati gati-karma kambojesv eva bhasito bhavati vikara evainam arya bhasante sava iti |*
> *hammatiḥ surastresu ramhatiḥ pracya-madhyamesu gamim eva tv aryah prayunjate |*
> *datir lavanarthe pracyesu datram udicyesu |*

That is: in Kamboja they call going savati; the Arya people call a vikara, that is a corpse, sava. In Surastra they call going hammati, and in the east-middle ramhati. In the east they call cutting dati, and in the north datra. Further on in that same Mahabhasya it is written: 'of one single word there are many apabhramsas; thus of the word go there are gavi, gauni, gota, gopotalika and the like.' Where these apabhramsa words were spoken is now hard to find; but there is no doubt that at that time also in Hindustan there were separate languages, and the spread of Sanskrit was only among the Arya people. Manu wrote the boundary of the country of the Arya people:

> *sarasvati-dradvatyor devanadyor yad antaram |*
> *tam deva-nirmitam desam aryavartam pracaksate ||*
> *(Manusmrti 2.17)*

Sabaean, Roman, Chinese, Tibetan and Basque (pp. 21-24)

The Sabaeans of Arabia Felix wrote letters for numbers in the manner of the south Semitic scripts. The Romans wrote I, V, X, L, C, D, M, and built other numbers by joining and taking away. The Chinese wrote from one to ten by strokes and then by separate signs; they also had a rod-numeral (suan-chou) method on a board, which is a place-method.

In Tibet the numbers from eleven to nineteen are made from the name of ten first and then the names of one, two; 20 from two and ten; the rest 21, 22-29 from the names of the ten and of the unit. In the same way further numbers also are made. To show numbers by the digits of the places is a very good and easy method; but who knows why people afterwards left this method. The later Aryabhata also in his book showed numbers in this same way (see my printed Mahasiddhanta). In the south in many places to this day they show numbers by the letters of the alphabet in the manner of the later Aryabhata. Many say that the people of Europe learned the method of writing numbers from Tibet. If this thing were so, then among them, for 11-19, everywhere, as in Tibet, first the word of 10 and then the words of 1-9 would come; and 21-29 also would be spoken as in Tibet.

In Basque, 1 = bat / badu, 2 = bi. In the Pingala books and kavya of the Hindi language also they take bi for two. Baba Dinadayalji in his Anuragabaga wrote this same bi.

Hindu, Arabic and Russian signs (pp. 25-26)

The Hindu digits are the nine signs and the sunya whose shapes, spoiling from the first letters of the number-words and from the old Brahmi marks, became the Nagari of today. The Arabs took these same digits from Hindustan and wrote them in two styles, the eastern and the western; from the western style the 1 2 3 4 5 6 7 8 9 0 of Europe were made. The Russians, after taking Christianity, used the letters of their Slavonic alphabet for numbers in the Greek manner, until the Hindu-Arabic digits came in.

Numerals of the time of King Asoka (pp. 26-30)

From the letters cut on the pillars of King Asoka, which in English rule were found and read by the very hard labour of the pandits of Europe, the guess of the pandits of Europe is that the oldest is the Brahmi script, which is from even before the time of Asoka. In the guess of Mr Buhler this Brahmi was current in Hindustan from 500 years before the Christian era. In the house of the kings of the Suryavamsa of Patna also, reading and writing was in this same script. Mr Buhler has also proved that this was made by the Brahmans; its root is not Semitic.

After the spread of this script, toward the Punjab, from a mix of some Arabic and some Brahmi, Kharosthi was made, which the Jain people call Kharodi. This, by the custom of Arabic, was written from the right side toward the left. (See Buhler's Indian Palaeography, and the plates of the Kalsi and other edicts.)

It appears that the Brahmans, from a mind of ninda, called the people of Arabia khara, that is donkey; therefore the script in which the words that came from the lips of those donkeys were written was called Kharosthi. Brahman people hold themselves napak (impure) in being touched by both donkey and Musalman; they become suddha by bathing with the clothes. By the influence of the Brahmans themselves Kharosthi sank, and Brahmi spread in every place.

The marks of the join toward the forehead in a man's skull the Hindu people call Brahma-lipi. In my judgement this Brahmi script was current among the Hindus thousands of years before Asoka, and it was made from those Brahma-letters of the skull; that is why its name became Brahmi. How the barakhadi was taught around Benares is written at the opening of the next pages.

End of the close translation of printed pages 1-30. Printed pages 31-61 follow in the next section (matras, Brahmi plate, speech-order, sunya, place-names, addition).

Purāṇic and ritual forms of the digits

The Purāṇikas hold that for worship they draw the digits in the shapes of the navagrahas. The homakārī Brahmins make the digits in the shapes of the kuṇḍas. From that one can learn a good deal. A paridhi is a short circle; a long paridhi can also be imagined, but the difference is only of length.

Paṇḍit Durgāśaṅkar, who knew gaṇaka-taraṅgiṇī, holds that the digits were drawn in the shapes of Kubera’s nine treasures. The names of those treasures are: padma, mahāpadma, śaṅkha, makara, kacchapa, mukunda, kunda, nīla and kharva. Dvivedī prints suggested outlines: a bud-like mark for padma = 1; a two-petalled form for mahāpadma? — rather, a flower of two petals for one reading of śaṅku/kunda types; three petals for nīla = 3; a tortoise-like loop for kacchapa = 4; a crocodile-like mark for makara = 5; a small lotus for kharva = 6; a fuller lotus for padma = 7; the greatest lotus for mahāpadma = 8; and a conch for śaṅkha = 9. The precise woodcut outlines of 1910 cannot be reproduced here; the point of the passage is that later Hindu tradition liked to see the nine digits as pictures of sacred objects.

If you think it over, all the world’s designs are only upright and slanting lines. Give a small child a pen and he begins to make upright or slanting strokes on the ground or on paper; sometimes he whirls the pen and a crooked closed circumference appears. From these lines all forms in the world have been made.

Changing the r of rekhā to l — as r, ḍ, l interchange in Sanskrit and Hindi — gives lekhā as another name of rekhā. Lekhā in the sense of ‘account’ has been current in India from before the Buddhists. In the lives of

many Prakrit authors who have commentaries by Buddhaghosa, the words lekhā, rūpa and gaṇanā all appear in the account of their schooling. Rūpa is still used in Sanskrit mathematics for the known coin or unit; Bhāskarācārya writes rūpa everywhere in that sense in his Bīja. Even today people say, ‘Bring the lekhā (account) of this.’

Jayarāma Jyotiṣī’s opinion is that from Pāṇini’s root likh (to arrange letters) first came lekhā, ‘that which is written,’ and by changing l to r one got rekhā. Bhānudīkṣita says the same in his commentary on the Amarakośa. From lines the signs of letters were made.

Jayarāma also exhibited one scheme in which the digits 1–9 are built from upright and slanting strokes (1 a single stroke; 2 a hook; 3 two hooks; and so on). One must not suppose that this particular combination occurred to everybody. In different times and countries writers combined upright and slanting lines in many different ways.

Etymologies of the Sanskrit number-words

Dvivedī next collects the traditional dhātu-etymologies of the number-names, chiefly from Pāṇini, the Uṇādi-sūtras, Hemacandra and Bhaṭṭoji Dīkṣita.

Eka. From iṇ (to go) with the Uṇādi affix kan, after elision of the nasal: that which is fit to be praised, or simply the going/unique one. Śrīharṣa, in Naiṣadha VII.63, compares Damayantī’s ear to the written form of the digit 9 (nava), which in his day had the curve of an ear.

Dvi. According to Pāṇini, from du (to go) with the affix i. According to Hemacandra, from ubhe (to fill). The Gujarati Śabdacintāmaṇi derives dvi from du (to split or to honour).

Tri. From tṝ (to cross water) with the Uṇādi affix ṭri: that which floats on water.

Catur. From cat (to ask, to go about) with the Uṇādi affix tur: that which searches on all four sides.

Pañca. From pañc (to spread) with the Uṇādi affix: that which is spread out.

Ṣaṭ. From ṣaṭ (to be in parts) with kip: that which has several members.

Sapta. From sap (combination, right knowledge) with kanin and vṛddhi: that by which knowledge is well obtained. Bhaṭṭoji in the Siddhānta-kaumudī explains samavāya as connection or right understanding.

Aṣṭa. From aś (to pervade, to heap) with kanin: that which is present everywhere.

Nava. From ṇu (to praise) with kanin: that which is worthy of a logician’s praise — or simply ‘new.’

When people did not yet know how to make paper they cut letter-forms on palm-leaves with an iron-pointed pen. To make the cuts visible they smeared the leaf with kakṣī (lampblack paste), which entered the grooves. From this smearing (lip) came the second name of letters, lipi. The word lipi in the sense of ‘script’ was already current before Kātyāyana, which is why on Pāṇini’s sūtra Kātyāyana made the vārttika yavanānī lipyām, proving yavanānī as ‘the script of the Yavanas’ — that is, the Greeks. There is no doubt that in Kātyāyana’s time Greeks were coming and going in India for trade. More will be said in the chapter on Greek numerals.

From lipi we still speak of Kāśmīrī-lipi, Devanāgarī-lipi, and the rest. Even now many people in Bengal and Madras take pleasure in writing on palm-leaves. From the root lip (to smear), by the sūtra ikaḥ kṛṣyādibhyaḥ, lipi is ‘that which is smeared.’ The Amarakośa says the same.

The old paṇḍits of Mīmāṃsā, Vedānta, Nyāya and Vyākaraṇa paid little attention to accounts. Therefore they punched one or two holes in the written palm-leaves and passed a strong thread through them, so that the leaves should not be shuffled; the leaves hung on the thread like beads of a rosary. Many such books are still in the libraries of the Benares Sanskrit College and the Asiatic Society of Bengal. Many books are also

written on the leaves of the tāḍipatra (Borassus) palm.

Even now, except for the jyotiṣīs, most paṇḍits of other śāstras cannot read or write a number above a hundred.

Near Chaukaghat in Benares, on the right bank of the Barna, there is an akhāṛā of Vedāntin sādhus. A great paṇḍit named Vaiṣṇavadāsa Bābā used to live there. My father read several Purāṇas with him, and I too read grammar with him. One day after the rains, in the sun of Citrā, he was airing his books. A gust scattered all the leaves of his Vyākaraṇa-Mahābhāṣya. From ten in the morning till evening he tried to restore the order by the sense of the text, and failed. I arrived for my lesson, found him dejected, and asked what was the matter. He said the book was ruined by the shuffling of leaves. I laughed and said, ‘Why be sad? I will set the leaves at once — are there not page-numbers on them?’ He said the numbers were there but he did not understand them. In half an hour I had the leaves in order. The Bābā was delighted and gave me a small reward of a fish-shaped ornament.

Do not suppose that the particular combination of slanting and upright lines which Jayarāma showed is the only form that occurred to anyone. Different writers in different countries mixed those lines in many ways.

The numerals of the Babylonian astronomers

The astronomers of Babylonia took a single wedge for one; two hands of wedges joined and set aslant for ten; and then ten-and-one, ten-and-two, ten-and-three, ten-and-four, twenty-three, thirty, and so on. They wrote a thousand by first showing ten, meaning ‘ten times a hundred.’ Ten thousand was made by placing ten again to the left of a thousand.

In Persia, in the time of Alexander the Great, letters were formed from these same wedge-shapes. In that alphabet aleph and beth have such forms; for the other letters see the German Alphabet orientalischer und occidentalischer Sprachen printed in 1859.

All the numbers so far found in Babylonia are below the decimal lakh. Two tables have also been found. The first writes the squares from 1 to 60. It may have been made between 2200 and 1600 years before the Christian era. In this table the places of the digits are sixty-fold, for 8^2 (=64) is written 1,4; likewise $9^2 = 1,21$; $10^2 = 1,40$; $11^2 = 2,1$.

Among the Hindus the same sixty-fold places are still current in planetary computation and in ṣaṭkāla. In Sanskrit 64 kalās are written 1;4. The digit of the highest place is written highest, and the lower below it.

Among Hindus the sixtieth part of a day is a ghaṭī, the sixtieth of a ghaṭī a pala, the sixtieth of a pala a vipala. The Āryabhaṭīya says in its section on time: ‘Sixty nāḍīs make a day; sixty vināḍikās a nāḍī.’

The current Sūryasiddhānta says that knowledge of planetary computation passed from the Sun to Maya the Daitya, and that Maya then spread that computation in the city of Laṅkā (see my commentary Subodhinī on the Sūryasiddhānta). Inquiry shows that Maya was a Greek. In Sanskrit the Greeks are called Yavana or Mleccha. Varāhamihira writes in the Bṛhatsaṃhitā:

> *The Mlecchas are Yavanas; among them this śāstra is duly established. They too are honoured like ṛṣis; how much more a twice-born who knows fate?*

Many hold that the Greek scholars Hypsicles and Ptolemy took this sixty-fold division (kalā, vikalā) from Babylonia to their own country, and that from there the same method entered India. However that may be, a solar year of 360 solar days is already written in the Ṛgveda:

> *Twelve fellies, one wheel, three naves — who understands this? In it are fixed together three hundred and sixty moving and unmoving nails. (ṚV 1.164.48)*

The second Babylonian table writes the measures of the bright part of the moon-disc from the end of amāvāsyā to the end of pūrṇimā. From the first to the fifth of the bright fortnight the bright measures are written 5; 10; 20; 40 (=80). Thereafter, on each tithi, the bright measure increases by sixteen. In this table the moon-disc = 240. For the first five tithis the bright part is in geometric progression; afterwards in arithmetic progression.

The Ṣaḍviṃśa Brāhmaṇa says that the moon has sixteen kalās; that through the fifth of the dark fortnight the gods eat five kalās, from the sixth to the tenth the Rudras eat five, from the eleventh to the fifteenth the Vasus eat five; on amāvāsyā the sixteenth kalā is produced in the moon, and by the favour of gods, Rudras and Vasus all the kalās return by pūrṇimā (Ṣaḍviṃśa Br. 4.6).

I think that, to count the fine bright portion, someone divided each kalā into sixteen parts, making the moon-disc = 256. Subtracting the sixteen parts of the sixteenth kalā, which remains hidden inside the moon, they fixed the visible disc at 256 − 16 = 240. ‘Moon-disc’ here means the diameter of the round moon seen at full moon.

To take the diameter of the lunar disc as 240 is the same as our own custom, in which Varāhamihira and others take the radius of a circle as 120 — that is, the diameter as 240 — and then state the jyā and koṭijyā of the arcs (see the Pañcasiddhāntikā).

In my view the first four numbers of the second table, 5, 10, 20, 40, are in sixteen-fold places, so that 5 = 5, 10 = 16, 20 = 32, 40 = 64. Sixty-four is the bright part of the fourth; 5 is the bright part of the first third of the bright fortnight. The bright part of the third tithi, 30 = 48, has dropped out of the table. Since moon = 240 and tithis = 15, the bright part in one tithi should be 240/15 = 16; therefore the increase on each tithi should be sixteen.

Egyptian numerals and words

On the walls of a kind of temple of Rameses III numerals have been found. From them it appears that the Egyptians of that time showed the places of numbers by various forms of birds’ heads. They had two kinds of signs: one for calculation and another for carving on pillars and the like. This is a matter of about 1200 years before the Christian era.

For calculation they had: 1 = a stroke, ua; 2 = two strokes, sen; 3 = three strokes, zemet; 4 = fuu or ofuu; 5 = zuau; 6 = sas; 7 = sefez; 8 = zemennu; 9 = paut or pest; 10 = a hobble or joined hands, mei; 20 = taut; 30 = mab; 40 = zement; 100 = a coil, saa; 1,000 = za; 10,000 = tüb; 100,000 = hefennu; 1,000,000 = heh; 10,000,000 = sennu.

For a lakh they used a seated man with both hands raised; for a crore, a circle. For hundred, thousand and ten thousand they used three kinds of birds’ heads. For ten, the two joined claws. If one takes the man as a water-creature and the circle as the egg or bone of some water-animal, then from ten upward all the places were conceived as water-things. Further on you will see that the same habit is found among the Hindus.

These people, like the Hindus, kept the units to the right and the higher places to the left and above. A specimen compound number in the 1910 plate reads, in Egyptian signs, 3,459. Above a crore no place-sign is found, and for a lakh there is no sign on the pillars.

On pillars some digits are found with a little mark before or after them. Their forms should be seen. Together with this one should read E. A. Wallis Budge, The Nile, and Books on Egypt and Chaldaea, vol. III. The Rev. G. Peacock, D.D., has written something on the forms of Egyptian numerals in his Arithmetic of that time.

The numerals of the land of Greece

Those who live in the land of Greece are called Greeks. They showed numbers by the letters of their own alphabet: α = 1, β = 2, γ = 3, δ = 4, ε = 5, ς = 6, ζ = 7, η = 8, θ = 9, ι = 10, κ = 20, λ = 30, μ = 40 … and so on, with special marks for 90 and 900. The same method is found in the books of the later Āryabhaṭa (see the Mahāsiddhānta).

Brahmins used to come to this country to save dharma, having crossed the Ṣaṭaka and Kaṭaka, and they still come. The Vedas have written that one should go beyond the Sarasvatī. The Dharmaśāstras say that without the sacrificial fire the morning and evening saṃdhyā should not be done.

As people began to search out all matters they came here. First the Arabs and the Greeks arrived. By mixing, the Hindus learned many things from them and they learned many things from the Hindus.

In the time of Alexander the Great, Greek merchants, paṇḍits and others all came to India. For two hundred years Greeks ruled in our north-western provinces and the Punjab.

Just as today hundreds of English words are current among Hindus and hundreds of Sanskrit-Hindi words among the English, so at that time Greek and Hindu words passed into one another.

In the Jaiminīya-nyāya-mālā, 1.1.6, it is written that the meanings of words such as pika may be taken either from village use or from mleccha usage. Mādhava explains that pika is the cuckoo, nema means ‘half,’ and tāmarasa means lotus. In Persian nem is ‘half,’ which is Jaimini’s nema. I asked several maulavīs here about pika and tāmarasa; they said those words are not found in Persian or Arabic. I asked several Europeans whether they might be Greek; no trace was found there either. That pika and tāmarasa are mleccha words is not in doubt, for otherwise Jaimini would not have written so. In Jaimini’s time those mleccha words were certainly well known; the matter is now so old that one cannot tell from which mleccha speech they came.

Kastūrī (musk), khalīna (bridle), ittham (in this way), milinda (bee), dīnāra (gold coin), maya, yavana, māṇittha and others came to the Hindus from the Greeks (see Gaṇaka-taraṅgiṇī).

India (saindhava), dvija (bhūrja, birch), pipar (pippali) and others passed from the Hindus through Persia, with some change of pronunciation, into Greek, even before Alexander. Karpūra (camphor), kuṣṭha (costus), nāla, deva and other Sanskrit words passed into Greek by mutual intercourse.

Hindu merchants first took peacocks to Bactria in Persia — which the people of that country in old times called Bāhlīka — to sell; the merchants there bought them and took them further, and people began to call the peacock the Bactrian bird. Among the Buddhists there is a Bāveru-jātaka, on which Buddhaghosa wrote a commentary in A.D. 5. The Bible also says that in Solomon’s time the Phoenicians brought peacocks from Ophir (Ābhīra) together with many other goods: ‘For the king had at sea a navy of Tarshish with the navy of Hiram: once every three years came the navy of Tarshish bringing gold and silver, ivory, and apes, and peacocks’ (1 Kings 10:22; 2 Chronicles 9:21). Hiram was king of Tyre in Phoenicia.

Mutual intercourse has gone on from very ancient times. In Patañjali’s Mahābhāṣya on Pāṇini it is written that in Kamboja they say śavati for ‘to go,’ while the Āryas say śava for a corpse; in Surāṣṭra they say hammati, in the mid-east raṃhati; in the east they say dāti for ‘to cut,’ in the north dātra. Further on the Mahābhāṣya says that each word has many apabhraṃśas: of go (cow) the corruptions are gāvī, goṇī, gotā, gopotalikā, and so on. It is now hard to find where those corruptions were spoken. But there is no doubt that even then there were many languages in India, and that Sanskrit was current only among the Āryas.

Manu wrote the boundary of the Āryas’ country: ‘The land between the Sarasvatī and the Dṛṣadvatī, created by the gods, they call Brahmāvarta’ (Manu 2.17).

From the time the Greeks came to India, phalita-jyotiṣa (astrology) spread here. In phalita many Greek and Arabic words are found (see the Bṛhajjātaka and the Nīlakaṇṭhī). Under the influence of phalita India has been so crushed that if, from today, people would turn their backs on phalita and look toward mathematics,

perhaps in thousands of years they might come level with Europe.

Such is the majesty of time: the country from whose dust-numerals the people of all countries have become and are becoming learned — the paṇḍits of that country themselves mingle with the dust, and yet day and night they are drunk with pride.

As among women here the influence of yantra-mantra is great, a hundred times greater is the influence of phalita-jyotiṣa among men and women. The mathematics on which phalita lives is being forgotten day by day. In English, people read mathematics only to pass the B.A. and M.A.; after passing, scarcely one in thousands talks of mathematics.

Phalita should be understood as a kṛtyā (a black art). In Europe too it stuck in the throats of Cardan, Kepler and others.

The numerals of the Sabaeans

The Sabaeans wrote 1 as a single stroke, 2 as two strokes, and so on in grouped strokes and bars, with special composite signs for 10, 20, 100 and 1,000. Arabic alif and nūn have related forms. The signs for 1–4 are of the same family as the Egyptian, the difference being that here they all stand between two pillars. The Sabaeans were in Arabia about A.D. 160.

Roman numerals

1 = I, 2 = II, 3 = III, 4 = IIII or IV, 5 = V, 6 = VI, 7 = VII, 8 = VIII, 9 = IX, 10 = X, 50 = L, 100 = C, 500 = D or a damaged form of D, 1,000 = CI or M. A bar over a numeral multiplies it by a thousand: Ī = 1,000, V̄ = 5,000, and so on. There is also another sign for 1,000. The first four numbers are made from the four upright fingers of one hand; five from the open hand; 6–8 by adding one, two, three; ten from two hands; nine by placing one to the left of ten. In some old inscriptions XX = 20, XXX = 30.

Chinese numerals

In China there are three kinds of numerals. In the first, one is written as a horizontal stroke; in the second and third, other simple strokes are used. The words are: 1 yi, 2 er, 3 san, 4 si, 5 wu, 6 liu, 7 qi, 8 ba, 9 jiu, 10 shi, 100 bai, 1,000 qian, 10,000 wan, and then higher compounds (yi, zhao, jing, gai, and so on). Dvivedī prints the common and the formal (dàxiě) characters side by side.

Number-words in Tibet

1 cig, 2 gnyis, 3 gsum, 4 bzhi, 5 lnga, 6 drug, 7 bdun, 8 brgyad, 9 dgu, 10 bcu. 11 is bcu-cig, 12 bcu-gnyis; 20 is gnyis-bcu; 21 gnyis-cig, and so on. From eleven to nineteen the numbers are made from the first syllable of ten plus the names of one, two … ; twenty from two and ten; 21–29 from the ten-word and the unit-names. All higher numbers are built in the same way.

To show numbers by the digits of the places is a very good and easy method; one does not know why later people abandoned it. The later Āryabhaṭa also showed numbers in this way in his book (see my edition of the Mahāsiddhānta). In the south, in many places, people still show numbers by the letters of the alphabet after the manner of the later Āryabhaṭa.

Many say that the people of Europe learned the method of writing numbers from Tibet. If that were so, then everywhere in Europe the words for 11–19 would be, as in Tibet, first ten and then 1–9, and 21–29 would be spoken in the Tibetan way. They are not.

Basque ‘one’ and ‘two’

In Basque, 1 = bat, 2 = bi. In Hindi pingala treatises and in kāvya, vi is also taken for two. Bābā Dīnadayāl in his Anurāga-mārga writes in one place cibikrī kī para (cibi = 2×2 = 4; cibikrī = a four-man palanquin). Vi occurs in many places in Hindi pingala. I think that from Sanskrit dvi, after the d dropped, vi remained, which later Hindi called vi.

Hindu numerals

The old Hebrew alphabet has twenty-two letters. They too showed numbers by letters. From aleph to teth the nine letters stood for 1–9; then yod = 10, kaph = 20, and so on. Their order of writing is the reverse of ours: 1 = aleph, 2 = beth, and 21 is written with the ten to the right of the unit. They had no sign for numbers above 400; higher numbers were shown by adding several numbers.

Arabic signs

The old Arabic alphabet also has twenty-two letters. From the abjad order they took numbers, as the Greeks did from letters. In course of time that alphabet grew to twenty-eight letters. This letter-numeral system became current about A.D. 800. In some places they show numbers after the Hindu fashion by letters. In their books numbers above 1,000 are also written by adding the letter-numbers. Wherever Semitic letters are current, numbers are shown by letters.

Russian signs

The Russian alphabet is made from Greek letters. In it there are thirty-six letters; by those letters numbers are shown. They have signs up to 10,000. Further numbers are made by adding a mark like a Greek accent to the letters. They have no number above ten crores. Peter the Great, in his time, introduced the Hindu numerals.

Numerals of the time of King Aśoka

Look at the plate: 200 is made from 100 and two, 300 from 100 and three, 400 from 100 and four, and so on; 2,000 from 1,000 and two, 10,000 from 1,000 and ten, 20,000 from 1,000 and twenty, 70,000 from 1,000 and seventy. If you look carefully you will see that in joining, the writer or the engraver has here and there let the forms of the joined numbers run together. The engraved numerals should be looked at again with care.

From seeing the numbers it is clear that in Aśoka's time there was no zero, nor the method of writing according to ten-fold places.

There is no doubt that everywhere counting on the fingers made the ten current, except in two or four places where the ten was less or more. Among fruit-sellers the gāṛī and the korī are current. In New Zealand there are eleven-fold places: they call 12 eleven-and-one, 13 eleven-and-two, and 22 two elevens.

In India, in speaking eleven, twelve … eighteen, the unit comes first and then the ten. That custom is not universal. In Latin eighteen is decem et octo = 10 + 8; in Greek oktō-kai-deka = 8 + 10; in French dix-huit = 10 + 8; in German achtzehn = 8 + 10. In Aztec eighteen is caxtolli-om-ey = 15 + 3.

I have already written that in Chinese san = 3 and shi = 10. Joining them they take shi-san for 10 + 3 = thirteen. That method is the reverse of the Hindus'. And they take san-shi for 3 × 10 = thirty, which agrees with the Hindu method. In Sanskrit one sometimes takes tridaśa for 30.

Merchants in some places, for understanding among their own people, speak numbers in other words. Thus the brokers of Benares say: 1 = sāğ, 2 = snān, 3 = ekvāī, 4 = nok, 5 = budh, 6 = ḍaik, 7 = pant, 8 = maṅg, 9 = kān, 0 = salāh, and so on.

Sanskrit number-words and signs

The vaiyākaraṇas hold that the word is beginningless like Parabrahman; therefore the letters, the parts of words, are also beginningless. Later the ṛṣis, to explain those letters to other people, made signs by which they could be shown in writing. People call those signs letters by courtesy; those letters are also called lipi (see pp. 9–10 of this book).

From the letters engraved in the pillars of King Aśoka, which in the English rāj were recovered and read by the great labour of the paṇḍits of Europe, the European paṇḍits infer that the oldest script is Brāhmī, older even than Aśoka's time. According to Bühler this Brāhmī was current in India from 500 years before the Christian era. In the houses of the Maurya kings of Patna writing was also done in it.

Bühler has also shown that this script was made by the Brahmins; its root is not Semitic. After this script had spread, toward the Punjab there arose, from a mixture of some Arabic and some Brāhmī, the Kharoṣṭhī — which the Jains call Kharoṣṭhī. It was written from right to left, in the Arabic manner. (See G. Bühler, On the Origin of the Indian Brāhma Alphabet, 2nd ed., Indian Studies III, Strassburg, Trübner, 1898.)

It seems that the Brahmins, in contempt, called the Arabs khara, 'ass'; therefore the books that issued from the lips of those asses, written in that script, were called Kharoṣṭhī. Brahmins think themselves impure (apavitra) if touched by an ass or by a Musalman; clothes become pure by washing. By the influence of the Brahmins, Kharoṣṭhī was suppressed and Brāhmī spread everywhere.

The join-marks on the human skull toward the forehead are called by Hindus Brahma-lipi. In the great old Sanskrit books it is written that whatever happiness or sorrow a man enjoys after birth is written on his forehead in Brahma-letters. Śrīharṣa writes in Naiṣadha I.16 that the Creator's lipi on the beggar's brow, 'this one shall be poor,' was not made false. Tulasīdāsa, in the dialogue of Rāvaṇa and Aṅgada in the Laṅkā-kāṇḍa, writes that looking at the skull one sees the digits written by Vidhi on one's own brow.

Therefore I am convinced that in the heart of some maharṣi the thought arose of making letters after the join-marks of the human skull, and that when they were made, being like the Brahma-lipi, the script was named Brāhmī. By making the Brāhmī script that maharṣi became in this world truly a second Brahmā, whose fame has spread unshaken through the whole world.

In Aśoka's time this Brāhmī had signs for twelve vowels, which were joined to the consonants. Hence in Prakrit it is called bārākhaḍī (= twelve-akṣarī = dvādaśākṣarī). In this Brāhmī a mātrā like an ear placed to the left or right on the head of a consonant was called in Prakrit kaṇṇa (ear), and the other mātrā, a vertical stroke, was called mālā or mattā.

These things have grown old; the forms of letters and mātrās have also changed; yet from our teachers we have read the bārākhaḍī in this way.

The order of speaking is the reverse of the order of writing

Separate signs had been made for 100, 200 … but the method of writing the larger number to the left of the smaller was the same as is current today in the maxim aṅkānām vāmato gatiḥ — 'the motion of digits is to the left.'

In the first copper-plate of Śīlāditya, 286 of the [Gupta/Śaka] era = A.D. 364; in the second, 356 Śaka = A.D. 434. In Dharasena's second plate, 272 Śaka = A.D. 350; in the fourth, 326 Śaka = A.D. 404 (Indian Antiquary, 2 February 1872).

From all this it is certain that the custom of aṅkānām vāmato gatiḥ had been current for a long time; but the method of showing any number by the nine signs 1–9 together with zero in the places of tens, hundreds … spread everywhere among the Hindus about Śaka 420. From that time the Śaka era also spread well everywhere. Āryabhaṭa did not write his date of birth in the Śaka era, but by that time the Śaka era had

become current in his region. Even before Āryabhaṭa the names of the ten, hundred and other places had been made. From Āryabhaṭa's 'in two nines of zeros, nine vowels' it is clear that zero had already been made before his time.

In a mirror the left part of the image that faces us lies toward our right side. In the same way, of a number written on the ground, on a board or on paper facing us, the left part will lie opposite our right: thus in 325, two is to the left of three and five to the left of two — as we face it.

This India is counted foremost in learning, and within it Benares is called the best seat of learning. Balarāma and Kṛṣṇa also studied at Benares with the ṛṣi Sāndīpani.

The people here, wiping out the smaller Sītā first and the greater Rāma afterwards, say Sītā-Rāma; yet in conduct they seat Sītā to the left of Rāma. The same is true of Rādhā-Kṛṣṇa and Gaurī-Śaṅkara.

At the very beginning of the Jaiminīya-nyāya-mālā-vistara, Mādhavācārya writes that in a royal assembly the honoured tapasvins and vipras should sit on the right; the śruti does not make the left the place of honour. That is, the greater should be seated on the right and the lesser on the left.

To seat a lakh-pati to the left of a crore-pati, a thousand-pati to the left of a lakh-pati, and so on, is counted honour. Therefore the digit of a lakh was placed to the left of a crore, the thousand to the left of a lakh, and so on; yet in speaking, as in Sītā-Rāma, Rādhā-Kṛṣṇa, Gaurī-Śaṅkara, people began to say first the unit and then the ten.

Munīśvara, in his Marīci commentary on verses 28–29 of Bhāskarācārya's Gaṇitādhyāya, quotes Kṛṣṇa Daivajña: in a line of seated men the former establishment of the higher is proper, and the places of those seated below them in due order are seen in the world to be the same. (For Munīśvara and Kṛṣṇa Daivajña see the Gaṇaka-taraṅgiṇī.) That is: seat the man of high rank in the rows first; seat those smaller than he after.

Zero (śūnya)

Śūnya is the sign of an empty place. When the decimal places were current and a place had no digit, something was needed to mark the emptiness, otherwise 15 and 105 and 150 could not be told apart. That mark is zero. In Sanskrit it is called śūnya, kha, ākāśa, bindu, viyat, nabhas and the like, and in verse-coding any word that means sky, hole, or emptiness may stand for 0.

The small circle or dot used for zero is, in Dvivedī's view, natural: a child whirling a pen already makes a closed circumference. The word śūnya is old in philosophy; its use as a numeral is later. The Brāhmī inscriptions of Aśoka, as already said, have no zero. By Āryabhaṭa's time (born Śaka 398 / A.D. 476) the nine vowels in 'two nines of zeros' show that zero was known. The decimal place-value method with zero spread generally about Śaka 420 (A.D. 498).

European writers long argued whether zero was Indian, Babylonian or Greek. Dvivedī's conclusion, argued from Indian inscriptions, the Āryabhaṭīya, and the later transmission to the Arabs, is that the zero as a place-holding digit in a decimal system is a Hindu achievement.

Where letters were formed

Letters and digits were first formed where writing materials existed: on birch-bark in the north-west, on palm-leaf in the east and south, on copper-plates for royal grants, and on stone for public edicts. The 'place of the formation of letters' in Dvivedī's heading is also the question of the homeland of Brāhmī. He sides with Bühler that Brāhmī is a Brahmin invention, not a borrowing of a Semitic alphabet, though Kharoṣṭhī in the north-west does show Semitic influence and a right-to-left ductus.

Where writing was done

Everyday calculation was done on a pāṭī — a board — dusted with powder, or on the ground with a stick, or with cowries, seeds and bits of bamboo. Such working leaves no monument. Only when the same numbers were cut in stone or copper did they survive. That is why Egyptian temple-walls still show numerals four or five thousand years later, while the ground-arithmetic of India has vanished. Paper came late; until then the book was a bundle of palm-leaves on a string.

Arabic numerals

The numerals which Europe calls Arabic are the Hindu numerals as they passed through the Arabs. In A.H. 156 (A.D. 773), in the reign of the second ʿAbbāsid caliph al-Manṣūr, an Indian astronomer went to Baghdad and said that a Hindu prince … [the 1910 text here recounts the well-known story of the arrival of a siddhānta at the caliph's court]. From that time — that is, from A.D. 773 — the Hindu method of writing digits and numbers spread well among the Arabs. In the western Islamic world these digits are called ghubār numerals (dust-numerals), because they were written in dust on a board.

The forms of 2, 3, 7 and 8 in the old Sanskrit digits, somewhat spoiled, are the ancestors of the corresponding Arabic and then European forms. In Germany about A.D. 1500 the printed forms were already close to those now used.

The Arabs themselves distinguished Hindu calculation (ḥisāb al-hind) from their older letter-numerals (abjad). Al-Khwārizmī's treatise, translated into Latin as Algoritmi de numero Indorum, carried the system into Europe. The very word algorithm preserves his name.

The opinions of Capenter and Harkel

Dvivedī reports the views of two European writers (the Hindi spellings Capentar and Harkel) who discussed the origin of the so-called Arabic digits. One line of European opinion still tried to derive the figures from Greek or Roman marks; another already admitted the Indian origin. Dvivedī's own position is firm: the place-value decimal digits, with zero, are Hindu; the Arabs were the carriers, not the inventors. Dust-writing on a board (ghubār) is the same practical method as the Indian pāṭī.

Sanskrit names of the places

In the śloka of Vālmīki the places named are daśa, śata, sahasra, ayuta, koṭi, śaṅkha, arbuda, madhya, antya, samudra, parārdha, padma, śaṅkha — so many places.

In Śaka 420 (A.D. 498) Āryabhaṭa wrote in the Gaṇitapāda of the Āryabhaṭīya:

> *Ekaṃ daśa ca śataṃ ca sahasram ayuta-niyute tathā prayutam |*
> *koṭy arbudaṃ ca vṛndaṃ sthānāt sthānaṃ daśaguṇaṃ syāt ||*

Here the place-names are eka, daśa, śata, sahasra, ayuta, niyuta, prayuta, koṭi, arbuda, vṛnda. Many hold that koṭy arbudaṃ ca vṛndaṃ is a corrupt reading, that vṛnda is idle here, and that the true reading is koṭy arbudam aravindam. Others hold that vṛnda here means lotus (aravinda = padma). On either view one more place, padma, appears. In my opinion vṛnda indicates still more places, because in 'kha-dvi-navake svarā nava' Āryabhaṭa takes eighteen places from two nines, which are still famous in Sanskrit arithmetic.

The ca at the end of the first pāda of this āryā is to be counted long by the rule of metre. Not knowing this, Dr. Kern needlessly showed a metrical fault (see his edition of the Āryabhaṭīya, 1874). Āryabhaṭa took niyuta for lakṣa. The Amarakośa also says vā lakṣā niyutaṃ ca tat.

The Ratnakośa writes the same places as Āryabhaṭa: śata, sahasra, ayuta, niyuta, prayuta, then feminine koṭi, arbuda — each ten times the one before.

In the commentary on verse 41 of the second chapter of Harṣa (the Saptaśatī), Nāgeśa quotes the Brahmāṇḍa Purāṇa:

> *Ekaṃ daśa śataṃ caiva sahasram ayutaṃ tathā |*
> *lakṣaṃ ca prayutaṃ caiva koṭir arbudam eva ca ||*
> *abjaṃ kharvo nikharvaś ca śaṅkha-padmau ca sāgaraḥ |*
> *antyaṃ madhyaṃ parārdhaṃ ca daśa-vṛddhyā yathākramam ||*

Here śaṅkha has dropped out and śabda-kha has come in its place; and from the last padma it seems mahāpadma is taken.

Śrīdhara in the Triśatikā writes: eka, daśa, śata, sahasra, ayuta, then lakṣa, prayuta, koṭi, arbuda, abja (lotus), kharva, nikharva, mahāsaroja (mahāpadma), śaṅkha, saritāṃ pati (ocean), antya, madhya, parārdha — these places, each ten times the former. Bhāskarācārya has also read these in his Līlāvatī.

The arithmetics of Lalla, Brahmagupta, Ṣaṭeśvara, Bhaṭṭa Balabhadra, Śrīpati and others are not found, but by tradition it is certain that in them too the names of the places will have been written in Śrīdhara's order.

That the last and greatest place is parārdha is not in doubt. Śrīharṣa also, in Naiṣadha III.40, takes parārdha as the greatest: if the three worlds were a court of counting, if life were equal to that court, if there were mathematics beyond parārdha, even then he who is to be counted would not be finished.

The names of the nine nidhis in Sanskrit books are: mahāpadma, padma, śaṅkha, makara, kacchapa, mukunda, kunda, nīla and kharva (Amarakośa, svarga-varga).

Ten, or the tens place

Daśa or dahāī is the first named place after the unit. It is the place of the fingers of two hands. All higher names are ten-fold steps from this one.

Hundred

Śata, Hindi sau or saikṛā, is ten tens. The word is Indo-European (Latin centum, Greek hekaton).

Thousand

Sahasra, Hindi hazār (a Persian word now current), is ten hundreds.

Ayuta, lakṣa (niyuta) and prayuta

Ayuta is ten thousand. Lakṣa (a lakh, 100,000) is called niyuta by Āryabhaṭa and the Amarakośa. Prayuta is a million (10 lakhs).

Koṭi, or crore

Koṭi is ten million, the Hindi crore. It is a principal commercial and astronomical unit.

Arbuda

Arbuda is a hundred million. Some later writers confuse it with arba/arab.

Abja (padma), kharva, nikharva and mahāpadma

These lotus-names continue the decimal ladder: abja or padma, kharva (a small lotus), nikharva (a somewhat larger lotus), mahāpadma (the greatest lotus). They are also names of Kubera's treasures. Wooden counters in these shapes — called goṭī in Hindi — may once have been used on the ground.

Śaṅku

Śaṅku, a stake or gnomon, is another high place-name in some lists.

Jaladhi (ocean), antya, madhya and parārdha

Jaladhi or samudra, antya, madhya and parārdha close the classical list. Parārdha, ‘half of the supreme,’ is the highest named place in the standard Līlāvatī series. What counters people used for ocean and the rest cannot now be known, because accounts made on the ground with lotus-seeds, cowries, bits of bamboo and bone last only a little while. If, as in Egypt, their forms had been cut in stone, they might still be present after four or five thousand years.

Arba (arab)

From Buddhist times the name arba also appears for one place of number. Many have taken arba as an apabhraṃśa of arbuda; but in my opinion those people took arba as the sea-horse. In Sanskrit arvā means horse. The Amarakośa gives vāji, vāha, arvā, gandharva and the rest as names of the horse. (From the root ah with the affix vanip one gets arvan.)

Śaṅkha and nīla

Śaṅkha is a famous sea-animal; its dry bone is also famous under the name śaṅkha. Among Hindus it is held very sacred; people sound it in the worship of the Lord. By nīla the seed of the blue lotus is taken. In the names of the places the names of gems — diamond, emerald, topaz — do not appear; therefore it seems that nīla does not mean the sapphire (nīlam).

Addition and subtraction of numbers

The method by which several numbers are gathered together — that is, added — has in Sanskrit the principal name saṅkalita or saṅkalana; in some places yoga and yukti also appear. Saṅkalana is from the root kal (to count, to tell) with the prefix sam: to count well, that is, to gather numbers.

When the places of numbers had been made, to gather the digits of each place, to carry the extra digits of a larger place into the digits of that larger place, and at the end to know the sum, is not a hard task. But how people actually did this work in practice is not in any Sanskrit mathematical book. Wherever the method of adding or subtracting is written, only this much is written: add or subtract in order — that is, from the units, tens … — or in reverse order — that is, first the digits of the highest place, then those of the next lower.

The later Āryabhaṭa writes in his Mahāsiddhānta: the making into one of many numbered things is saṅkalita; that which is taken away from the whole is vyavakalita, and the remainder is śeṣa.

Bhāskarācārya also writes in the Līlāvatī: the sum should be made in order of the digits, or the difference as the places require. But from saying only so much, the mind cannot see how the act is to be done. That is why in mathematics, medicine, music, and the art of moving and making weapons, people say that in these sign-arts there is a knot whose secret remains with the guru.

Addition and subtraction in sixty-fold places

In jyotiṣa the places of kalā and vikalā are sixty-fold. Addition and subtraction are then the same as in decimal places, except that one carries or borrows at sixty instead of at ten. Hindus still add and subtract ghaṭīs, palas and vipals in this way. The same sixty-fold arithmetic is the Babylonian inheritance already described.

Ancient addition and subtraction

Before place-value, addition was the gathering of separate tally-marks — strokes, wedges, cowries — and subtraction the taking away of so many marks. The Egyptian and Sabaean stroke-groups show this stage. The Roman system, with its subtractive IV and IX, is a later refinement of tally-writing. True ease in addition and subtraction comes only with place-value and zero.

Multiplication and division

Because in the ‘hills’ (pahāṛe, multiplication tables) the numbers go on being doubled, tripled … perhaps the name pahāṛ (mountain) was given, for mountains too appear one higher and greater than another. Since Hindu counting has ten-fold places, the tables of the digits 1–9 run from the once-times to the ten-times. Teachers, to make children firm in accounts, have them by heart the tables up to forty. The Romans have twelve-fold places, therefore in Europe the tables of 1–12 contain all the multiples of 1–12.

Before the English rāj, so that much practical arithmetic might be done by heart, in India the tables of paunā, savaiyā, ḍeḍhā, aḍhāī, hūṭā, dhaucā and paucā were committed to memory. Even now banias certainly teach their children paunā, savaiyā and ḍeḍhā.

In paunā the multiplier is ¾; in savaiyā 5/4; in ḍeḍhā 3/2; in aḍhāī 5/2; in hūṭā 7/2; in dhaucā 9/2; in paucā 11/2. Thus 4 paune = 3, 4 savaiyā = 5, 4 ḍeḍhe = 6, 4 aḍhāī = 10, 4 hūṭe = 14, 4 dhauche = 18, 4 pauche = 22. In some places there is also a vikaṭ pahāṛā in which a number is taken 1¼, 1½, 1¾, 2¼, 2½, 2¾ times: for example, ‘a bit more than a bit more is savā do (2¼),’ and so on.

There is also ek-ekkā, in which the squares of 1–100 are said: ekā ek, dūā dū cār, triṭikā nav, cār chok 16, and so on. There is also ek-baṛā gyārahā, in which the numbers 11–20 are multiplied up to 11–20 times: gyārah gyārah 121, gyārah bārah 132, gyārah terah 143 … ; bārah gyārah 132, bārah bārah 144, bārah terah 156 … ; bīs gyārah 220, bīs bārah 240 … . In the Mithilā country there is also a baṛā paunā in which the numbers are ¾ × 1½ times, and so on.

Paunā, savaiyā, ḍeḍhā, aḍhāī are clearly the apabhraṃśas of the Sanskrit pādona, sapāda, adhyardha and adhyardhādi. But that hūṭā, dhaucā and paucā are the apabhraṃśas of adhyardha-tri, adhyardha-catuḥ and adhyardha-pañca is a little doubtful; perhaps the Sanskrit words were first spoiled in Prakrit and those Prakrit words further spoiled into Hindi hūṭā, dhaucā and paucā.

Since tahsili schools have been opened in village after village, the custom of teaching paunā, savaiyā … has died; now children are only taught a few tables. Hindus had gone on being very skilled in algebra, but it is a matter of great sorrow that algebra has been removed from the tahsili schools. Now no saṃskāra of it is produced in children’s minds; therefore when they grow up, how will they be able to think of bīja?

In Sanskrit there are six methods of multiplication. In A.D. 1540 Gaṇeśa showed a seventh method in his commentary Buddhivilāsinī on Bhāskara’s Līlāvatī, but that method is not current. In this country the Musalmans call this method zarb-koṭhrī. Many hold that the Arabs invented it. They call it shabaka (network).

The method of the zarb-koṭhrī: taking the places of multiplicand and multiplier as equal base and upright … [Dvivedī then sets out the lattice or gelosia multiplication, the kapāṭa-sandhi or ‘door-junction’ method, the direct place-by-place method, and the other classical pāṭī methods].

Division is the exact reverse of the first method of multiplication, kapāṭa-sandhi. Sanskrit astronomers learn that first method of multiplication well, so that the act of division may be understood at once. After learning it they multiply much faster than those who learn the school’s current (fourth) method. The fault of that current method is only this: if a mistake is made in the middle, the work must be done again from the beginning, because all the digits are rubbed out as one goes and new ones written in their place; therefore one cannot find where the mistake occurred.

In the Manoramā commentary on Bhāskara's Līlāvatī and in the book of Lucas de Burgo there are several new methods of division, but they are not worth mentioning; in truth they are only other forms of the old method.

Checking the product and the quotient

The famous method of checking the product and the quotient from the digit 9 — that is, of knowing whether these two have come out right or wrong by casting out nines from the sum of the place-digits — has been written by the Arab astronomers in their books, and sometimes people also call it the 'Hindu proof.' But except for the later Āryabhaṭa's Mahāsiddhānta it is not discussed in any other famous Sanskrit book. Nārāyaṇa Paṇḍita in his Gaṇita-kaumudī writes a method of checking the product from any chosen number:

> *The product of the remainders of multiplicand and multiplier after division by a chosen number should equal the remainder of the product after division by that number; if they match, the multiplication is correct.*

In Europe Lucas de Burgo wrote this method in his arithmetic for addition, subtraction, multiplication and division. He also wrote a method from 7, but in that method one must divide the numbers by 7. A better method is from 11. In it one takes the difference of the sums of the odd and even place-digits after division by 11, or one subtracts each place-digit from the next, and so on to the end; the last remainder is the remainder on dividing the number by 11. If a digit will not subtract from the next, add 11 and then subtract. Example: in 247896, 4−2=2, 7−2=5, 8−5=3, 9−3=6, 6−6=0, so 247896 is divisible by 11. In 283471, 8−2=6, which will not go into the next 3, so add 11: 3+11−6=8, 4+11−8=7, 7−7=0, 1−0=1; therefore on dividing 283471 by 11 the remainder is 1.

Gerbert also wrote this method in his books. He was a very famous man. His pupil King Otto III made him Pope under the name Sylvester II. He died in A.D. 1003. The method is not new; it is drawn from the old foundation (see verse 57 of the Chāyādhikāra in my edition of the Brāhmasphuṭasiddhānta).

Close translation of the 1910 text, printed pages 31 to 61

The pages that follow render, in order, the Benares edition from the end of the old school-drill of the matras through the first working of addition. They are the next thirty pages of Sudhakara Dvivedi's book. Glyphs that the 1910 woodblocks no longer show clearly are described in words.

How the matras were taught (p. 30 continued)

The old drill of the matras, still used by gurus around Benares when boys learn to put the vowel-signs on consonants, is a string of Prakrit scraps of Sanskrit:

ka bin kanne ka | kannun ka | resom ki | dirgo ki |
tare ku | bare ku | ek mat ke | do le ke | kan mat ko |
dumati kana ko | maste kam | dasi kah ||

That is: bin kanne = vina karnena, without the ear-stroke; kannun = karnena, with the ear-stroke; resom = hrasva, short; dirgo = dirgha, long; tare = tale, below; bare = from vardhakya, long by the matra; ek mat = ekamatra, with one mora; do le = dve matre adaya, taking two morae; kan mat = karnena matraya ca, with one ear-stroke and one mora; dumati kana = matra-dvayena karnena ca, with two morae and one ear-stroke; maste = mastake, a bindu above the head of the letter; dasi = daksine (aksarasya), two bindus on the right. All these are worn-down Sanskrit words. Around Benares the gurus still teach boys to set the matras in this way. At Patna, Darbhanga and elsewhere the words of the barakhadi drill differ a little, but the root of all of them is the Sanskrit just given.

On the day a Sanskrit-reading boy around Benares begins his letters, the guru first has him write 'sri-ganesaya namah' and learn to utter it. The gurus also make the boys learn by heart the first sloka of Valmiki's Ramayana:

ma nisada pratistham tvam agamah sasvatih samah |
yat kraunca-mithunad ekam avadhih kama-mohitam ||

The boy who first recites it from memory is taken to be the one who will soon become a pandit. There is a saying that when Vyasa began to make the Mahabharata, Ganesa alone was ready to write; therefore, holding him the greatest of scribes, they have the boys write his name first of all and cause them to worship him.

Boys who go to a village school to a guru to learn Hindi, Kaithi and the like are first made to write 'o na ma si ddham'. That Buddhist custom still runs. It is an apabhramsa of 'om namah siddham'. Patanjali already wrote in the Mahabhasya:

mangalik acarya mahatah sastr'-oghasya
mangalartham siddha-sabdam aditah prayunkte |
pasyati tv acaryo mangalarthas ca
siddha-sabda aditah prayukto bhavati ||

'The auspicious teacher, for the welfare of the great flood of sastra, uses the word siddha at the beginning.' 'The teacher sees that the word siddha, used at the beginning, is for auspiciousness.' Now, wherever the tahsili schools have been opened, the old custom is falling away.

Brahmi before Asoka, and the Kalsi alphabet (pp. 31-33)

In my judgement this Brahmi script was current among the Hindus thousands of years before Asoka, and it was made from the Brahma-letters of a man's skull; that is why its name became Brahmi. At that time the Sanskrit vowels r, ri, l, li will also have been in it; but just as today, for want of use, they are dropped in Hindi, Kaithi and the rest, so in Asoka's time they were dropped in Prakrit. In the Saradatilaka the letters are taken as deities, and meditations on the letters are written, but between those meditations and the actual

shapes there is no match at all.

How could the Brahmi of Asoka's time, thousands of years after Brahmi was first made, remain in its original shape? Even on pillars of Asoka's own time, ten or twenty years apart, differences of shape are found in the Brahmi; what then is to be said where thousands of years lie between?

The alphabet of Asoka-period Brahmi that was made from the Kalsi inscription is set out on the plate. A stroke to the right of the head makes ka; a further stroke makes ki; a longer one makes ki-long. Below the foot to the right, or down in the foot, one writes ku or the like. A mark to the left of the head makes ke; marks on both sides, the left stroke a little off the right, make ko or kau. A small mark above the head to the right makes kam.

Raja Sivaprasad, in his Bhugol-hastamalaka, by a slip wrote this same Brahma-letter as 'Pali letters'. In that alphabet there are mistakes in several places as well. If, to straighten the line, one puts a short slanting stroke above the head of each Brahma-letter, a great likeness to the letters of today's Devanagari appears at once.

How the written digits grew from the Brahma-letters (p. 33)

According to the Brahmi of Asoka's time, the Sanskrit words eka, dvi, tri and the rest yield the digits as follows. One is the spoiled shape of the first letter of eka. Two is from dvi; three from tri; four from catur; five from panca; six from sad; seven from sapta; eight from asta; nine from nava. Looking at the shapes it is clear that the spoiled forms of the Brahma-letters of eka, dvi, tri and so on, changing little by little, are today's Sanskrit 1, 2, 3 and the rest.

The spoken number-words from eleven to ninety-nine (pp. 33-35)

In Sanskrit: 11 ekadasa, 12 dvadasa, 13 trayodasa, 14 caturdasa, 15 pancadasa, 16 sodasa, 17 saptadasa, 18 astadasa, 19 una-vimsatih, 20 vimsatih. From Panini's grammar one-and-ten becomes ekadasa, two-and-ten dvadasa, six-and-ten sodasa, eight-and-ten astadasa. But 'nava ca dasa ca navadasa' Panini did not make. For nineteen Panini makes two forms: ekena na vimsatih, ekanne-vimsatih or ekonavimsatih. By saying 'one short' from above one also takes the shorter word una-vimsatih for 19. In the Yajurveda, navadasa is also used for 19 (see Yajurveda-samhita, chapters 9 and 14).

In English too, in speaking 11-19, as in Sanskrit dvi, the unit comes first and then the ten. In speaking 19 the Vedic custom has been taken. In Sanskrit, except for navadasa, una-vimsatih has been used. For 20, Sanskrit and English have one and the same speech-habit, whose sense is 'two tens' (twenty).

21 ekavimsatih, 22 dvavimsatih, 23 trayovimsatih, 24 caturvimsatih, 25 pancavimsatih, 26 sadvimsatih, 27 saptavimsatih, 28 astavimsatih, 29 una-trimsat, 30 trimsat. Here too one first says the unit and then the ten. In the Veda, navavimsatih occurs for 29. In English speech the habit is the reverse of this. In speaking 30 both sides have one habit: 'three tens' (thirty).

So also 31 ekatrimsat through 39 una-catvarimsat, 40 catvarimsat; 41 ekacatvarimsat through 49 una-pancasat, 50 pancasat; 51 ekapancasat through 59 una-sasti, 60 sasti; 61 ekasasti through 69 una-saptati, 70 saptati; 71 ekasaptati through 79 unashiti, 80 asitih; 81 ekasitih through 89 navasitih, 90 navati; 91 ekanavati through 99 navanavati.

From Panini's grammar 49 becomes ekanne-pancasat / ekona-pancasat. In English the speech-habit is the reverse of the Sanskrit, but in the words for 30, 40, 50, 60, 70, 80 and 90 the habit is one and the same. On both sides the words were made in the manner of three tens, four tens, ... nine tens.

The Prakrit words for 1, 2, 3 and the rest are worn-down forms of the Sanskrit words written above, and from Prakrit, further spoiled, have come the Hindi words for 1, 2, 3 current today. In Sanskrit 10 is also called pankti. By Panini's sutra 5.1.59, pankti, vimsat, trimsat, catvarimsat, pancasat, sasti, saptati, asiti, navati,

satam — these words are rudhi, that is, ready-made.

Speech-order the reverse of writing-order (pp. 35-39)

Leaving aside the method of the later Aryabhata, all the later jyotisis write numbers in the reverse of the order of speech. That is: in speech, in dvadasa, one first says two and then ten; but in writing one first writes the ten and after it the unit, '12'. In this way, writing the digits of units, tens, hundreds from the right toward the left to show a number — this method is not found in the Vedas. In the Vedas, numbers above a hundred are written in several pieces.

> *trisata na sankavo 'rpitah sastir na cala-cala |*
> *(Rgveda 1.164.48)*

Here the number 360 is read in two pieces, 300 and 60. In the Yajusa Jyotisa-vedanga the number 366 is read in two pieces, 300 and 66. The number 1830 is read in the form of a product of 366 and 5 (Yajusa Jyotisa, verse 28).

In Somakara's bhasya, wherever there is a statement of Garga, numbers above a hundred are everywhere spoken in two pieces: pancatrimsat ca astadasa-sati = 30 + 1800 = 1830; caturvimsati satatmakam = 24 + 100 = 124; dvatrimsat satam = 32 + 100 = 132; astadasa-sati sasty-adhika = 1800 + 60 = 1860; dvadasa satam = 12 + 100 = 112; dasadhika dve sahasre = 10 + 2000 = 2010 (see my printed Somakara-bhasya).

From Valmiki's Ramayana: 'dasa-varsa-sahasrani dasa-varsa-satani ca' — 10 x 1000 years + 10 x 100 years, that is 10,000 + 1,000. From the Mahabharata, Striparvan, chapter 26, verse 9:

> *dasayutanam ayutam sahasrani ca vimsatih |*
> *kotyah sastis ca sat caiva ye 'smin rajan rane hatah ||*

In this, 10 x 10,000 + 10,000 + 20 x 1,000 + 60 x 10,000,000 + 6 x 10,000,000 is read — that is 100,000 + 10,000 + 20,000 + 600,000,000 + 60,000,000 = 660,130,000. In this way the number is written in six pieces.

Aryabhatiya, Ganitapada, verse 10: 'caturadhikam satam astagunam dvapancasat tatha sahasranam' — here 62832 is read as 8 x 104 + 62 x 1000 = 832 + 62000 = 62832. In 'sasty abdanam sastir yada vyatitas trayas ca yuga-padah' the number 3600 is read as 60 x 60 = 3600; but in the bhaganas, 'kuyu-ghrn' and so on, first the smaller number and then the larger is read. That same method is among the Sinhalese, but there a number larger than 9999 cannot be written with their twenty signs. They have no zero either.

In the Manusmrti, 24 is written from 'try-astavarsah' / 'astavarsa' in the manner of 3 x 8. In Bhattotpala's commentary on the Brhatsamhita, the quotations of the Paulisa and Surya siddhantas all read numbers in the order of the places eka, dasa, sata. Lalla, Varahamihira, Brahmagupta, Balabhadra and the rest all write digits by today's well-known rule ankanam vamato gatih. The later Aryabhata, in the patiganita chapter of his work, also wrote numbers by this same rule ankanam vamato gatih (see my printed Mahasiddhanta). In the Jaimini-sutra numbers are taken by letters, but all are written by today's well-known leftward motion.

The copper-plates of the kings of Valabhi that have been found, in which mixed letters of Kharosthi and Brahmi appear, have Saka dates all written by the method ankanam vamato gatih. The only difference is that, there being no zero, separate signs had been made for 10, 20, 100, 200 and so on; but the method of writing the larger number to the left of the smaller was the same that is current today as ankanam vamato gatih.

On Siladitya's first plate: Saka 286 = A.D. 364. On another: Saka 356 = A.D. 434. On Dharasena's: Saka 272 = A.D. 350; Saka 326 = A.D. 404. (See Indian Antiquary, vol. I, p. 14, etc.)

From all this it is certain that the custom of ankanam vamato gatih had been current for a very long time; but the method of showing a number by the nine signs 1-9 alone, with zero standing in the places of ten, hundred and the rest, spread everywhere among the Hindus about Saka 420. From that same time the Saka era

also spread well everywhere. Aryabhata did not write his time of birth in the Saka era, but by then the Saka era had become well known in his region. Even before Aryabhata the names of the places dasa, sata and so on had already been made. From Aryabhata's 'kha-dvinavake svara nava' it is plain that zero had already been made before his time.

The seating of the high and the low, and why the places run left (pp. 38-40)

In a mirror, the left part of the image that is seen in front stays toward our right part. In the same way, the left part of a number written on the ground, on a board or on paper in front of us will stand opposite our right part — as in 325, two is to the left of three and five to the left of two.

That land which is counted foremost in learning — even in it Benares is called the best seat of learning. Balarama and Krsna too studied at Benares with the sage Sandipani.

atha gurukule vasam icchantav upajagmatuh |
kasyam sandipanim nama hy avanti-pura-vasinam ||
(Bhagavata Purana 10.45.31)

The people here speak first the smaller Sita and after her the greater Rama, saying Sitarama; but in behaviour they seat Sita to the left of Rama. The same is so in Radhakrsna, Gaurisankara and the rest. At the very beginning of the Jaimini-nyaya-mala-vistara, Madhavacarya has shown:

raja-sabhayam ete tapasvinah pujya vipra daksinam bhagam upavesaniyah |
ete ca bhrtya vama-bhaga iti kramam karoti ||

That is: the great should be seated on the right and the small on the left. To seat a millionaire to the left of a crore-pati, a thousand-pati to the left of a lakh-pati, and so on, is itself counted honour. Therefore the digits of a lakh were placed to the left of a crore, of a thousand to the left of a lakh; but in speech, as in Sitarama, Radhakrsna, Gaurisankara, first the unit and then the ten came to be spoken.

Munisvara, in the Marici commentary on verses 28-29 of Bhaskara's Ganitadhyaya, has written the saying of Krsna Daivajna:

abhyarhita-sthanasya panktau purva-nivesas tad-arvaks-sthana-sthitinam
savya-kramena sthapanam ucitam lokesu tatha drsyate ||

That is: to seat first in the row the one of high rank, and to seat those of lower rank to his left, is also seen in the behaviour of people. (For Munisvara and Krsna Daivajna see the Ganaka-tarangini.)

Zero (p. 40)

The shape of sunya, that is of the empty, was made empty like the sky: 0. For in Sanskrit, from however many words mean sky, sunya is taken. In grammar, from 'sunah samprasaranam' and the lengthening, both sunya and sunya are made. In the Amarakosa it is written 'sunyam tu vrsikam tuccha-riktake' — that is, sunya is in the sense of empty. Therefore the Arabs translated sunya into Arabic as sifr. This sifra became Latin zephirum; then from zephirum, in English, zero; or, straight from the Arabic, cipher.

For the larger countings, by counting on the fingers of both hands one ten at a time, the name of ten tens was made sai-kada (a hundred), because by doing so the word does not grow in speech. In place of 'ten tens', saying sau gets the work done with one word. In the same way, by counting again and again on the ten fingers of the hands, separate names — ten, twenty and so on — were kept for each ten.

Where the letters were formed (pp. 40-41)

From Panini's Siksa it is clear that Mahadeva made the letters ('trisastir va catuhsastir varnah sambhu-mate matah'), and Mahadeva's remaining is for the most part at Benares; therefore Benares is called Kailasapuri,

Visvanatha-nagari, Anandavana, Kasi, Mahasmasana and so on. Vyasa too, in his old age, came to Benares. Agastya lived at Benares; later, on the great entreaty of the gods, he went outside. Ganesa dwells in Kasi itself. Taking Benares as chief, Gautama Buddha also came here first.

For Sanskrit learning Benares is the chief place. For the teaching of Rama and Krsna, Ugrasena, Vasudeva and all the Yadavas thought the Kasi-trained sage Sandipani the best. From all this it appears that some pandit of Benares first spread this science of the digits. From here the people of Arabia took this science to their own country, and, having understood the method well, in honour of Hindustan they began to call these digits hindisa and the method hisab-i hind. Later the merchants of Europe took them from Arabia to their own country and began to call these digits Arabic notation — the numeral-signs of the Arab people.

Where writing was done (pp. 41-42)

At first people scattered dust on the ground and did mathematics upon it. Wherever you look in the old siddhantas, reckoning is written as done on the ground itself. In all the old books reckoning is written as dhuli-karman. Bhaskaracarya, in the commentary on verse 4 of the lunar-eclipse chapter of his Grahaganita, has 'atra dhuli-karmana pratyaksa-pratiteh'. In Bhaskara's books dhuli-karman comes in several places. Bhaskara also wrote of doing mathematics on a phalaka; that is why, in the yantra chapter of his Goladhyaya, he named one instrument the phalaka-yantra itself. To spread dust or abir on a phalaka (a board or a small board) and do mathematics upon it — this custom has been current in Hindustan from a very old time. Bhaskaracarya also gives news of writing on the ground with chalk. In the parilekha he writes 'khatikaya rekha ucchadya' — that is, having drawn lines with chalk (dudhi), do the working.

From this it is clear that people also wrote on boards with dudhi. At the very beginning of the Buddhists' Kataha-jataka, in the story of a seth's son, the school of study, the phalaka and so on are mentioned. Alberuni, in A.D. 1030, in his description of the schools of Hindustan, wrote that in the schools he saw boys writing with a white thing on a black board.

To spread battu, dust or abir on a board and do reckoning upon it — this method was still used in the Benares Sanskrit College in the time of my own studies. Later Bapudeva Sastri introduced the English slate. Now a hanging wooden blackboard or a slate is current. For the most part reckoning is now done on paper with a pencil.

The numerals of the Arabs (pp. 42-45)

The old Arab people also showed numbers by the letters of the alphabet. With them: 1 alif, 2 ba, 3 jim, 4 dal, 5 ha, 6 waw, 7 zay, 8 ha, 9 ta, 10 ya, 20 kaf, 30 lam, 40 mim, 50 nun, 60 sin, 70 ayn, 80 fa, 90 sad, 100 qaf, 200 ra, 300 shin, 400 ta, 500 tha, 600 kha, 700 dhal, 800 dad, 900 za, 1000 ghayn. Beyond a thousand they had no further signs. Their numbers are very like those of the Sabaeans; only the shapes of the letters differ.

In A.D. 622, when Muhammad fled from Mecca to Medina, the people of the country of the Semites, whom at first no one knew, began to play a strange play in the drama of this history. The people of separate tribes all joined with one another, became one in one faith, and in the heat of religion became one jati. They all began to go about on every side with swords in their hands. In a few days Syria and Mesopotamia came wholly under them. By their power Persia and Hindustan both entered the kingdom of the Saracens. North Africa and nearly all of Spain also came into their power; but in western Europe, by the power of Charles Martel, in A.D. 732 they were stopped from going further. At that time the rule of the Muslims had spread from Hindustan to Spain.

In the end a very great quarrel arose among themselves for the caliphate. In A.D. 755 the kingdom of the Muslims was split in two. One sat as caliph at Baghdad, and the other at Cordova, which is in Spain.

This spreading of the Arabs is a matter of great wonder; a greater wonder still is with what ease those people left their wandering wild habit, took a settled habit, and set their rule over great learned men. In all the countries they had won they spread the Arabic language well.

In the rule of the Abbasids a new time of the history of learning began toward the east. Baghdad is settled on the bank of the Euphrates; it lies between Hindustan and Greece; to its east is Hindustan and to its west Greece. Therefore the Arab people began to learn new things from both the Greeks and the Hindus.

It was in the fortune of the Arabs that, in a time of trouble for the kings of Greece and of Hindustan, they should save the sciences of those lands from disappearing, and afterwards hand those sciences over to the people of Europe. The Arabs made very little progress of their own in mathematics. Of the things they had already learned from Greece and Hindustan they perhaps found the trace of one or two new things. Their mind did not settle on new matters of science; their power of thought was small; but they were very clever in trade. The matters of science on whose thought the Greeks and the Hindus were glad — on those the Arabs made no thought at all. Their mind never went to the conic sections of the Greeks, nor to the kuttaka and varga-prakrti of the people of Hindustan.

In A.H. 156 (A.D. 773), in the reign of the second Abbasid caliph al-Mansur, an Indian jyotisi went to Baghdad and began to say that, at the word of a Hindu prince, a table of this science had been made. The Arab pandits wrote the name of that prince as Fighar (or the like). Of the line of that same Vyaghramukha king with whom Brahmagupta stayed, some 'Fighar' (vyaghra) will have been; there is no doubt of this; and the jyotisi who went to Baghdad will have been someone from among the pupils of Brahmagupta. From the book of Abu Ma'shar, the jyotisi of Balkh, it is clear that whatever count of the motion of the planets is in Brahmagupta's book, that same is also in that table.

At the word of the caliph al-Mansur, Muhammad ibn Ibrahim al-Fazari made a translation of it into Arabic and gave it the name Sindhind or Hind-sind. From that same time, that is from A.D. 773, the Hindus' digits and method of writing numbers spread well in Arabia. In the western Arab lands these digits are called ghubar numerals. In Arabic, ghubar means dust. I have already written that among the Hindus the first custom was to scatter dust on the ground and do reckoning upon it. Therefore, when the Arab people in Hindustan saw the digits on the dust, they gave them this very name: the digits-on-dust.

The views of Cantor and Hankel (pp. 45-46)

Cantor and Hankel have shown, from the old Sanskrit number 2578, today's English 2578 in this way: old Sanskrit forms; then the same a little spoiled in writing; then eastern Arabic; then western Arabic ghubar digits; then the eleventh century; the thirteenth century; the sixteenth century. If you look at the Brahma-letters it will be clearly known that the spoiled shapes of the first letters of dvi, panca, sapta and asta themselves are the old Sanskrit 2, 5, 7 and 8.

In Germany, until A.D. 1700, only the Roman digits — such as are on a clock — were current; but from A.D. 1482 itself the Hindus' method of writing numbers had also been started. From the sixteenth century of the Christian era the Hindus' method of writing numbers spread well through all the educated countries.

Sanskrit names of the places (pp. 46-52)

The second mantra of the seventeenth chapter of the Yajurveda-samhita:

dasa ca dasa ca satam ca satam ca sahasram ca sahasram cayutam
cayutam niyute ca niyutam ca prayutam cabudam ca nyarbudam ca
samudras ca madhyam cantas ca parardhas caita me agna istaka dhenavah santv amutramusmin loke |

In this the names of these places come: dasa, sata, sahasra, ayuta, niyuta, prayuta, arbuda, nyarbuda, samudra, madhya, anta, parardha. In the Bhasya-vadadipa on this, Mahidhara writes that a hundred multiplied

by ten becomes a thousand; a thousand multiplied by ten becomes an ayuta; an ayuta multiplied by ten becomes a niyuta; niyuta is a lakh; a niyuta multiplied by ten becomes a prayuta; a prayuta is ten lakhs, and the taking of prayuta is an indication of a koti; a prayuta multiplied by ten is a koti; a koti multiplied by ten is an arbuda; an arbuda multiplied by ten is a nyarbuda; by the word nyarbuda the number abja is to be known. The taking of these is an indication of the numbers named kharva, nikharva, mahapadma, sankha and so on, which Mahidhara has in mind after padma. In this, koti, kharva, nikharva, mahapadma and sankha are extra. Mahidhara was later than Bhaskara; therefore, taking them from his Lilavati, he wrote these extras.

In the fifteenth and sixteenth sutras of the fourth anuvaka of the fifth kanda of the Atharvaveda-samhita the words for 1-11 and for 20, 30, 40, 50, 60, 70, 80, 90, 100 and 1000 come. In the same way in the Rgveda also the words for 10, 100, 1000 and so on come.

Valmiki-ramayana, Kiskindhakanda, sarga 38, verses 30-31:

sataih sata-sahasrais ca vartante kotibhis tatha |
ayutais cavrta vira sankhamibhis ca parantapa ||
arbudair arbuda-satair madhyais cantyais ca vanarah |
samudrais ca parardhais ca harayo hari-yuthapah ||

Sarga 39, verse 29: 'tatah padma-sahasrena vrtah sankha-satena ca / yuvarajo 'ngadah praptah pitus tulya-parakramah.' And sarga 48, verse 12, of a ten-year-old vanara son in the forest. In the library of the Benares Sanskrit College there is a book written in Bengali letters on teriya (bhojapatra) leaves, by estimate more than four hundred years old, in which the reading is: sataih sata-sahasrais ca kotibhis cayutair api / prayutair agamisyanti sankhamis ca parantapa — and so on. In that reading, in the place of sankha the reading is saha. That reading also seems right to me. From the time of the Buddhists the name sankha has been added.

From these slokas Valmiki spoke these places: dasa, sata, sahasra, koti, sankha, arbuda, madhya, antya, samudra, parardha, padma, sankha.

In Saka 420 (A.D. 498), in the Ganitapada of the Aryabhatiya:

ekam dasa ca satam ca sahasram ayuta-niyute tatha prayutam |
koty-arbudam ca vrndam sthanat sthanam dasagunam syat ||

In this the names of these places come: eka, dasa, sata, sahasra, ayuta, niyuta, prayuta, koti, arbuda. Many hold that in this sloka 'koty-arbudam ca vrndam' is a corrupt reading, that vrnda is here useless, and that the pure reading is 'koty-arbudam aravindam'. Many hold that vrnda here means the lotus (aravinda, padma). However that may be, on both views one more place, padma, comes; but in my judgement, by vrnda Aryabhata showed that there are still more places. For in 'kha-dvinavake svara nava' he took, from dvi-navaka, eighteen places, which to this day are well known in Sanskrit arithmetic.

The ca at the end of the first pada of this Arya will be counted long by the method of a verse-book. Not knowing this, Dr Kern showed in it a needless fault of broken metre (see his edition of Aryabhata, 1874). Aryabhata took niyuta as lakh. In the Amarakosa also it is written 'va laksa niyutam ca tat.' In the Ratnakosa also Aryabhata's own places are written: satam sahasram ayutam niyutam prayutam tatha / sati kotir arbudam iti kramad dasa-gunottaram.

In the commentary on verse 49 of the second chapter of the Durga (Saptasati), Nagesa has written a statement of the Brahmanda Purana: eka dasa satam caiva sahasram ayutam tatha / laksam ca prayutam caiva kotir arbudam eva ca / abjam kharvo nikharvas ca sankha-padmau ca sagarah / antyam madhyam parardham ca dasa-vrddhya yathakramam. In this nyarbuda has dropped out and lakh has come in its place, and from the last padma a mahapadma seems to have been taken.

Sridhara in the Trisatika: eka dasa satam asmat sahasram ayutam tatah param laksam / prayutam kotim atharbudam abjam kharvam nikharvam ca / tasman mahasaroja sankha saritam patim tatas tv antyam / madhye parardham ahur yathottaram dasagunah samjnah. In this the places are: eka, dasa, sata, sahasra, ayuta, laksa, prayuta, koti, arbuda, abja (lotus), kharva, nikharva, mahasaroja (mahapadma), sankha, saritam pati (samudra), antya, madhya, parardha. These same Bhaskaracarya also read in his Lilavati.

The arithmetics of Lalla, Brahmagupta, Vatesvara, Balabhadra, Sripati and the rest are not found, but by tradition it is certain that in them too the names of the places in order will have been those written by Sridhara. That the last and greatest place is parardha, there is no doubt. Sriharsa too, in Naisadha 3.40, has taken parardha as the greatest:

yadi triloki-ganana-para syat tasyah samaptir yadi nayusah syat |
pare parardhe ganitam yadi syat ganeya-nihsesa-guno 'pi sa syat ||

In the Sanskrit books the names of the nine nidhis are: mahapadmas ca padmas ca sankho makara-kacchapau / mukunda-kunda-nilas ca kharvas ca nidhayo nava (Amarakosa, kharga-varga). In this, padma, sankha, nila and kharva come. In the current Sanskrit books I have not found 'nila' among the names of the places. It is possible that in some Buddhist book, taking it as a nidhi of Kubera, someone also kept the name nila for one place.

From the time of the Buddhists, after a thousand, not many new names came; and so that even one place might increase, 'ten' was joined with each of the names; ayuta, prayuta, samudra, madhya, anta and parardha were left; in the place of niyuta the short name laksa was taken; and three new names — nila, arba and sankha — were added.

Today around Benares these names of the places are current: ekai, dahai, saikada, hajar, das-hajar, lakh, das-lakh, karod, das-karod, arb, das-arb, kharb, das-kharb, nil, das-nil, padma, das-padma, sankh, das-sankh. Somewhere, in the place of das-hajar and so on, they say dah. All these have come into Hindi by way of Prakrit. All are worn-down forms of Sanskrit words. Thus from dasa, dah, then dahai; from sata, sau, then saikada; from sahasra, sahassar, then, the first letter falling away, hassar, and from hassar, hajjar and hajar. In the same way all the others are apabhramsas of Sanskrit.

In Europe the word million was first shown by Pacioli in A.D. 1494 in the Summa de arithmetica. In Italy ten tons of gold are called a milione; from that the word also became current as a place of number. It is a wonder why billion, trillion, quadrillion, quintillion, sextillion, septillion, octillion and nonillion — so many places — were made; for among the people of Europe there is no such great number as the circle of the sky or the solar days of the life of Brahma. It is possible that, seeing the Sanskrit places, someone spread the places far in his own country as well. In 1580, Jean Trenchant of Lyons made milliard current for a thousand millions. Million and the rest had been current in Chuquet from A.D. 1484.

In arithmetic the greatest number is the circle of the sky. Brahmagupta wrote its measure in his Goladhyaya: 'ambara-yojana-paridhih sasi-bhagannah sunya-kha-kha-jinami-gunah.' From this the yojanas of the sky-circle = 18,712,080,864,000,000. Brahmagupta too will have taken it by tradition from the books of the old people (it is possible he took it from the Visnudharmottara Purana). In the opinion of Paulisa the yojanas of the sky-circle = 18,861,777,091,200 (see p. 90 of my printed Brhatsamhita with Bhattotpala's commentary).

A number still greater than the yojanas of the sky-circle is the solar days in the full life of Brahma: 1000 x 2 x 360 x (360 x 1000) x ... = 111,974,400,000,000,000? [Dvivedi's figure is the classical count of savana days in a Brahma-life]. In the sky-circle, digits come as far as the madhya place, and in the solar days of the life of Brahma they come as far as the parardha place; therefore it appears that eighteen places and their names were made in Sanskrit. Later people added one more place and made nineteen places.

Ten, or the tens place (pp. 52-53)

In Sanskrit the god of wealth is taken to be Kubera. Kubera's place is Kailasa (ke jale lusati sobhate iti kailasah) — that is, in water. He first also lived in Lanka; from fear of his younger brother Ravana he fled from Lanka. Because of living in water he became master of all water-things. His nine nidhis — mahapadma (the great lotus), padma (lotus), sankha (the well-known sea-creature), makara (the well-known water-creature, the crocodile), kacchapa (the well-known water-creature, the tortoise), mukunda (a flower), kunda (a flower of the month of Magha), nila (the blue lotus), kharva (the small lotus) — these are all water-things. If you look and think, the root of trade is wealth itself; therefore, to keep behaviour running, people, taking Kubera as lord of wealth, took one by one a water-thing from among his nine nidhis and set them in the places of the digits.

There is a fragrant water-grass which people today call pani ka sotha or nagar-motha. The root of that same grass, which is like kaseru, was set in a place, and that place became well known from the short name of that grass, dasapura. (Dasapuram dasapuram chavanam jivitahvayam — so Vacaspati.) People, not taking the long name, began to call it dasa, which today in Hindi is famous as the name dahai.

If you take the shape of the root of this grass to be such-and-such, then one such root = 10, two = 20, three = 30, and so on. Or, from the root damsi (to bite), dak = that which bites; in that sense they may have made a shape by joining the fingers of both fists in the place of a ten, as the people of Egypt set a mark in the place of ten.

Hundred, or the saikada (p. 53)

In the place of this rank people first set a piece of bamboo. Then, when in Sanskrit this bamboo received the name sataparva (sataparva yava-phalo venur maskara-tejana — Amarakosa 2.4.161), its short name sata was taken. Suppose the shape of one piece is such-and-such; then one piece = 100, two pieces = 200, and so on.

Thousand, or hajar (p. 53)

It appears that in this place people first set durva-grass. Afterwards, when in Sanskrit durva received the name sahasra-virya (Amarakosa 2.4.158), people began to call this place sahasra. Suppose its shape is such-and-such; then one = 1000, two = 2000, and so on.

Ayuta, laksa (niyuta) and prayuta (pp. 53-54)

In Sanskrit, laksa or laksa means lac, which is produced from the lac-insect. Bhanu Diksita, in his commentary on the Amarakosa, writes that laksyate iti laksa; from the root laks (to see) with the suffix ghaj, and then by the prsodaradi rule, laksa is made. Laksya is also in the sense of a mark. From all this it appears that laksa is a kind of pellet from the mixing of lac and another colour, which, from the root yu (to mix) with the suffix kta and the prefix ni, you may also call niyuta (a mix of several colours).

When this pellet is of one colour only, that is, unmixed, you may call it ayuta; and when the mix of colours is even more than a niyuta, you may call it prayuta. Therefore a pellet of plain lac colour became famous as ayuta, a pellet of a mix of two colours as niyuta, and a pellet of a mix of three or four colours as prayuta. Or cowries of one colour, two colours and three or four colours became famous by those names. If you call the ayuta-pellet la-1, the laksa-pellet la-2 and the prayuta-pellet la-3, then la-1 = 10,000, two such = 20,000, and so on; la-2 = 100,000; la-3 = 1,000,000.

In the southern countries, even now, in doing mathematics, cowries are current here and there in the places of the digits. Those people set cowries of several different shapes in different places. From this it is guessed that the old people, taking them as sea-things that last many days, had set cowries of different colours in the places of ayuta, laksa (niyuta) and prayuta. Even now gamblers, each keeping his own cowrie of a different shape, play at dice.

Koti, or karod (pp. 54-55)

Near the sea there is a Brahmi creeper or grass. Its Sanskrit name is koti-varsa. In its juice there is a taste like honey; therefore Bhanu Diksita, in his commentary on the Amarakosa, has given its derivation: kotibhir meghair hanti ... madhu ya sa koti-varsa — that is, she who rains sweetness from her own edges is koti-varsa. It is much sown in Lanka; therefore its other name is lankopika (that which is sown in Lanka). The old people set its root in a place and began to call that place by its short name koti. Suppose the shape of its root is such-and-such; then one = 10,000,000, two = 20,000,000, and so on.

Arbuda (p. 55)

Arbuda means a fleshy lump. In the Medini-kosa it is written: 'arbudo mamsa-kile 'stri paruse dasa-kotisu | mahidhara-visese na ...' From mamsa-kila it appears to be the bone of some sea-creature. Just as the gomati-cakra, the shell, the coral and so on — the bones of sea-creatures — are held sacred, so this too will have been held the sacred bone of some creature. The place on which it was set became famous by the name arbuda. If its shape be such-and-such, then one = 100,000,000, two = 200,000,000, and so on.

Abja (padma, lotus), kharva (small lotus), nikharva (a somewhat larger lotus) and mahapadma (the largest lotus) (pp. 55-56)

From padma an ordinary lotus, that is the seed of the lotus, was taken. The third name after this, mahapadma, is a very great lotus, that is the seed of a very great lotus; therefore from the kharva and nikharva between lotus and great-lotus, a kharva-lotus (small lotus) and a nikharva-lotus (a somewhat larger lotus) have been taken. For speaking quickly the half-names kharva and nikharva have been taken.

There are many kinds of lotus. From their seeds a mala is made. That mala of seeds is called kamalaksa. In Sanskrit, kharva means the small. In the Amarakosa it is written 'kharvo hrasvas ca vamanah'. Just as in yu, with the prefix ni, niyuta was made, so by putting the prefix ni on all, nikharva was made. By putting a prefix on roots the sense of the roots is changed; therefore Diksita wrote in the Siddhanta-kaumudi: 'upasargena dhatv-artho balad anyatra niyate / hara-ahara-samhara-vihara-pariharavat.' From all this it is clearly known that from padma, kharva, nikharva and mahapadma four kinds of seed of four kinds of lotus were taken. The places in which these seeds were set also became famous by these same names.

Sanku (pp. 56-57)

Sanku is a water-creature. In the Amarakosa it is written: 'timingiladayas catha yadamsi jala-jantavah / tad-bhedah sisumaraudra-sankavo makaradayah.' The place in which they used to set the bone of this creature, people began to call sanku. Suppose the shape of this bone is such-and-such; then one = 10^12, two = 2 x 10^12, and so on.

Jaladhi (samudra), antya, madhya and parardha (pp. 57-58)

In the sea all jewels remain; its very name is ratnakara. Therefore ten times a sanku people called samudra, that is sagara; ten times that, antya, that is the end of the sea (the beginning of the great sea); ten times that, madhya (the middle of the great sea); and ten times that, parardha (the second half after the middle of the great sea). Suppose the shape of samudra is such-and-such, of antya such-and-such, of madhya such-and-such, and of parardha such-and-such; then those marks stand for 10^14, 10^15, 10^16, 10^17.

In this way, if you give attention to the names, it is clearly known that, to understand the places dasa, sata and the rest, the old Hindus took different water-things (just as the people of Egypt set the beaks of water-dwelling geese). The only difference between the two is that the Hindus took the very pieces, bones, roots and seeds of those things, and the people of Egypt, not taking the things themselves, made their shapes. Afterwards, when the method of writing numbers was started, those places began to be spoken by the names of those same things. It is possible that, like the pieces of chess, they made wooden shapes of lotus and the

rest, which in Hindi are also called saudhi.

What they set in the place of samudra and the rest, it is now impossible to find out; because the reckonings that were done on the ground with lotus-seeds, pellets, cowries, pieces of bamboo, bones and so on can remain on the ground only a short time. If, as in Egypt, their shapes had been cut on stone, it was possible that today, four or five thousand years having passed, they would still be present, as in Egypt the cut digits are still present in many places.

Arba (arab) (p. 58)

From the time of the Buddhists the name arba has also come for one place of number. Many have taken arba as an apabhramsa of arbuda; but in my judgement those people took from arba the sea-horse. In Sanskrit, arva means a horse. In the Amarakosa it is written 'vaji-vahah arva gandharva-haya-saindhava-saptayah.' (From the root r with the suffix vanip, arvan is made.)

Sankha and nila (pp. 58-59)

Sankha is a well-known creature of the sea; its dry bone is also famous by the name sankha; among the Hindus it is held very sacred; people sound it in the worship of Bhagavan. From nila the seed of the blue lotus was taken. Among the names of the places the names of jewels — hira, panna, pokhraj and the rest — have not come; therefore from nila it appears that nilam (sapphire) was not taken.

Addition and subtraction of digits (pp. 59-61)

The method by which several numbers are gathered, that is added, has in Sanskrit the chief name sankalita or sankalana; but somewhere the names yoga and yukti also come. Sankalana or sankalita is made from the root kal (of sound and of number); by putting the prefix sam, it is to count well, that is to gather numbers.

When the places of the numbers had been made, to gather the digits of each place, to take out from them the digits of a higher place and to join those to the digits of that higher place, and at the end to know the sum — this work is not hard; but how people did this work in behaviour is not in any Sanskrit mathematical book. Wherever a method of adding or subtracting is written, only this much is written: in order, that is units, tens and so on, or in reverse order, that is first the digits of the greatest place and then the digits of the place below it, go on joining in addition and taking away in subtraction.

The later Aryabhata wrote in his Mahasiddhanta:

sankhyavatam bahunam ekikaranam tad eva sankalitam |
yad apastam sarvadhanat tad vyavakalitam tu sesakam sesam ||

Bhaskaracarya also wrote in his Lilavati: 'karyah kramad utkramato 'thava ankair yogo yatha-sthanakam antaram va.' But from saying only this much, the thought how the working is to be done does not come into anyone's mind. Therefore in mathematics, medicine, music and the science of making and running weapons, people are in the habit of saying that in these sign-sciences there is a key; without the guru that secret does not open.

We show one example of how, by guru-parampara, we do this working. Suppose 325, 1785, 952 and 25 are to be added. Scattering dust on the ground or on a board, writing the digits under each place, one has the columns. Now by the order-method one first adds the digits of the units: 5 + 5 + 2 + 5 = 17. One rubs out the five of the top row and writes 7 in its place, and says 'one came to the hand'. Then one adds this one with the digits of the tens. Here this sum is 18. Rubbing out the two of the tens of the top row, one writes the eight of the tens-sum 18, and says 'one came to the hand'. Then one adds this one with the digits of the hundreds. Here this sum is 20. Rubbing out the three of the hundreds of the top row, one writes the zero of the hundreds-sum 20 in its place, and says 'two came to the hand'. Then one adds this two that came to the hand with the digits of the thousands-place. Here this sum is three. Writing this in the thousands-place with the digits of the top

row, the sum = 3087 was made.

By this same method one always writes the digits of the sum in the top row. In the reverse-order method one first adds the digits of the greatest place and, rubbing out the first digit in the top row, writes there. Thus here first the one of the thousand will be set in the top row; then one will write the nine of the hundreds-sum 19 in the place of the three of the top row, having rubbed that three out, and will join the one that came to the hand with the one of the thousand in the top row, and, rubbing out that one, will set two. In this way one goes on. In this reverse working the digits of the top row have to be rubbed out several times; therefore the Sanskrit jyotisis (ganakas), in behaviour, always make the sum by the order-working.

Now today many people draw a slanting line under the lowest row and write the digits of the sum under that line, as the boys of the schools write. The Sanskrit ganaka cannot add on paper, because those people have to rub out the digits of the top row and write new digits in their place.

This speech that has come down by tradition — 'one came to the hand, two came to the hand' — makes it clearly plain that for the places of ten, hundred, thousand and the rest the old people used to keep by them many roots of grass, pieces of bamboo and so on for reckoning; and in addition, as many as were needed were taken into the hand. As in the example above, in the sum 17 of the unit-digits, 7 was set in the place of the unit, and for one ten one root of nagar-motha was taken in the hand. Then, this root being joined with the roots of the tens, of the 18 roots that were made, 8 were set in the place of the tens, and the ten roots were set aside and in their place one piece of bamboo was taken in the hand. In my judgement, though the old custom has gone, the old speech has not risen and gone. Everywhere 'came to the hand' or 'struck the hand' is current.

End of the close translation of printed pages 31-61 of the 1910 Benares edition. The next original stretch is printed pages 62-91 (sexagesimal addition, ancient addition, multiplication and division, square root).

Close translation of the 1910 text, printed pages 62 to 91

These pages continue the last instalment without a break: subtraction as the ganakas still do it on dust, sixty-fold addition and subtraction, the old counters of pebble and chalk, the multiplication tables of the Hindi school, the six Sanskrit methods of multiplication, kapata-sandhi, division, and the opening of square root and fractions.

A doha of Tulasidasa, and subtraction (pp. 62-63)

Tulasi-pati rati anka sama sakala sadhana suna |
anka-rahita kachu hatha nahi anka-sahita dasa-guna ||

Tulasidasa wrote this in the Dohavali: all means without the beloved are empty, as a place without a digit; with the digit they are ten-fold. For subtraction both the order-method and the reverse-order method are written, but the Sanskrit ganakas subtract only by the reverse-order method.

They subtract in this way. Suppose 9683 is to be taken from 12781. The viyojya (that from which one subtracts) is set above, the viyojaka (that which is subtracted) below, each in its place:

12781
9683

In subtracting they speak thus: 'Nine will not go from two; therefore make the next one into sunya, ten came; ten and two are twelve; nine went from twelve, three remained' (rub out 12 and set 3 in its place). 'Six went from seven, one remained' (rub out 7 and set 1). 'Eight went from eight, sunya remained' (rub out the upper eight and set zero). 'Three will not go from one; therefore make the one of the upper hundreds into sunya, and make the sunya into nine; ten came; ten and one are eleven; three went from eleven, eight remained.' By doing this the remainder (sesa) = 3098.

In this way, when the lower digit will not go from the upper, they lessen by one the place-digit at its side, rub out that larger place-digit and write it so, add ten to the upper place-digit, subtract the lower place-digit from that sum, and, rubbing out the upper place-digit, write the remainder-digit in its place.

In the order-working the method is the same; the only speciality is that the subtracting begins from the unit. But in it the upper digits have to be wiped several times; therefore no jyotisi subtracts by that method. All people learn the reverse-order method well.

This subtracting is called in Sanskrit vyavakalita or vyavakalana. These too are made from the root kal (of sound and of number). By putting the prefixes vi and ava their sense is to separate. Viyoga and antara also come in the sense of subtracting.

Addition and subtraction of sixty-fold place-numbers (pp. 63-64)

Where the places are sixty-fold — that is, amsa, kala, vikala and prativikala, or day, ghati, pala and vipala — the Sanskrit jyotisis add in this way. (I have already written that the Sanskrit jyotisis work sixty-fold places.) When the sum of a lower place reaches sixty or more, they carry one to the next higher place and write the remainder below sixty in that lower place, rubbing out what stood there.

In subtracting, if the vipala, pala and so on of the viyojaka are greater than those of the viyojya, they lessen by one the digit of the upper place, subtract the viyojaka's vipala, pala and so on from 60, and add the remainder to the viyojya's vipala, pala and so on.

Example: from 3 days, 17 ghatis, 20 palas, 21 vipalas take 1 day, 19 ghatis, 22 palas, 27 vipalas. They begin and speak: '27 will not go from 21; therefore make 20 into 19, sixty came; 27 went from sixty, 33 remained; 33 and 21 are 54' (rub out 21 and set 54; rub out 20 and set 19 there). 'Then 22 will not go from 19; therefore make 17 into 16, sixty came; 22 went from sixty, 38 remained; 38 and 19 are 57' (rub out 19 and set

57; rub out 17 and set 16). In this way they go on up. Jyotisis often begin the working of subtraction from the upper place: here they first subtract 1 from 3, then lessen this remainder by one and bring 60, subtract the viyojaka's 19 from this, add the remainder 41 to the viyojya's 17, rub out that 17 and set the sum in its place. In this way they go on down.

Ancient addition and subtraction (pp. 64-67)

In old time digits were understood by pebbles, pieces of stone or wooden counters. Among village people addition is still done with pebbles. The tamoli people still, by putting dots of lime, understand that so many packets of pan have been made. Reckoning was also done on a board with dust or battu spread, with the help of counters. In Egypt and Greece the same custom obtained.

Herodotus writes that the Greeks, in writing and in reckoning, take their hand from left to right, but the people of Egypt from right to left. The old counters of Egypt — the stroke for one, the heel-bone for ten, the coil for a hundred, the lotus for a thousand — have already been shown in the chapter on Egyptian numerals. When the sum of a place reached ten of its own marks, those ten were taken away and one mark of the next place was set.

The same is the sense of the Hindi speech 'came to the hand'. The old people did not write a carry; they picked up a counter. When paper and the printed book came, the counter became a spoken one, and the school-boy writes the carry under a line. The Sanskrit ganaka on dust still rubs and writes in the top row, because that is the old counter-method without the counters.

Among the Romans the same adding of I, X, C, M by grouping and replacing is the ancestor of their written numerals. Among the Chinese the counting-rods on a board, and later the suanpan, do by beads what the Hindu did by motha-root and bamboo. Dvivedi notes the kinship and returns to the Indian working.

Multiplication and the pahare (pp. 67-70)

In this the digits go on increasing two-fold, three-fold and so on; perhaps for this reason the name pahar (parvata, mountain) was kept, because mountains too are seen one higher and larger than another. In the Hindus' counting, the places being ten-fold, in the pahare of the digits 1-9 the digits remain from one-fold to ten-fold. Gurus make the boys learn the tables by heart up to forty, to make them firm in reckoning. Among the Romans the places are twelve-fold; therefore in Europe, in the tables from 1 to 12, all digits remain 1-to-12-fold.

Before English rule, so that many reckonings of behaviour might be done by word of mouth, in Hindustan pauna, savaiya, dedha, adhaiya, hutha, dhauncha and pauncha were learned by heart. Even now banias certainly teach their boys pauna, savaiya and dedha.

In pauna the number remains 3/4-fold; in savaiya 5/4; in dedha 3/2; in adhaiya 5/2; in hutha 7/2; in dhauncha 9/2; in pauncha 9/4-fold. Thus 4 paune = 3; 4 savai = 5; 4 dedhe = 6; 4 adhai = 10; 4 huthe = 14; 4 dhaunche = 18; 4 paunche = 22. Somewhere there is also the custom of the vikat pahare. In it the number remains 3/4, 5/4, 3/2 and 5/2 fold; for example dyodhe dyodhe = sava do (2 1/2); dyodh adhai = paune car (3 3/4). There is also an ekanna; in it the squares of 1-100 remain. They are spoken ekanna ek, dua du car, tritika nau, car chok 16, and so on.

There is also a great gyaraha. In it the numbers 11-20 remain, multiplied up to 11-20 times. Thus gyarah gyarah 121, gyarah barah 132, gyarah terah 143; barah gyarah 132, barah barah 144, barah terah 156; bis gyarah 220, bis barah 240. In the Mithila country there is also a great pauna; in it numbers remain 3/4-fold, that is paune.

Pauna, savaiya, dedha, adhaiya are clearly worn-down forms, in order, of the Sanskrit words padona, sapada, adhyardha and adhyardha-dvi. But there is some doubt that hutha, dhauncha and pauncha are

worn-down forms of adhyardha-tri, adhyardha-catuh and adhyardha-panca. Perhaps the Sanskrit words were a little spoiled in Prakrit, and then those Prakrit words, further spoiled, became in Hindi hutha, dhauncha and pauncha.

Since the tahsili schools were started village by village, the custom of teaching pauna, savaiya and the rest has risen and gone. Now some pahare are taught to the boys. The Hindu people had gone on being very skilled in algebra; but it is a matter of great sorrow that algebra has been taken out of the tahsili schools. Now the samskara of it is not even born in the boys' minds; therefore when they are grown, how will they be able to think of bija?

The six Sanskrit methods of multiplication (pp. 70-74)

In Sanskrit there are six methods of multiplication. Ganesa, in A.D. 1540, showed a seventh method in the Buddhivilasini, his commentary on Bhaskara's Lilavati, but that method is not much current. In this region the Muslim people call this method zarb-kothri. Many hold that the Arab people invented this method. Those people call it shabaka.

The method of the zarb-kothri: taking a lattice of cells equal to the places of the gunya (multiplicand) and the gunaka (multiplier), one writes the gunya along the top and the gunaka down the side, fills each cell with the product of its row and column (the tens of that little product in one half-cell, the units in the other), and adds by the slanting files. This is the well-known grating or lattice method.

Kapata-sandhi. As in a door one leaf stays a little mounted upon the other leaf that is shut, so in this method the unit of the multiplier remains under the digit of the highest place of the multiplicand; therefore its name kapata-sandhi (the joining of the door-leaves) was kept.

The digit of the highest place of the multiplicand is multiplied, by the method of the pahare, by the digit of each place of the multiplier, and is set on the head of the place-digits of the multiplier; but the digit that is made on multiplying by the unit-digit of the multiplier is written in the place of the highest place-digit of the multiplicand, that digit being rubbed out. When something comes to the hand it is joined with the digit at its side, that digit is rubbed out, and the sum is written in its place. Then the multiplier is lifted and set in the same way under the next-highest place-digit of the multiplicand, and the working above is done. When the digits of all the places of the multiplicand have been multiplied, the product stands in the place of the multiplicand. This working cannot be done on paper; even so, for understanding, an example is shown.

Let the gunya be 2576 and the gunaka 345. To know the product one first writes the gunya and under it the gunaka. Multiplying 2 by each place-digit of the multiplier and setting the multiplied digits by the method above, one then shifts the multiplier so that its unit sits under the second place-digit 5 of the multiplicand, and the digits of all the places are set in order to the left, and the same working is done again. Then again the multiplier is shifted and its unit is set under the 7 of the multiplicand. When the digits of all the places of the multiplicand have been multiplied, the product will be in the top row.

Aryabhata, in his Aryabhatiya, which was made in A.D. 499, took this method as well known and left it. Brahmagupta, in the ganitadhyaya (arithmetic) of his Brahmasphuta-siddhanta, which was made in A.D. 628, made no mention of it, taking it as well known. Sridhara in his Trisatika wrote the name of this method kapata-sandhi and kept the name of the product pratyutpanna. Brahmagupta also wrote the name of the product pratyutpanna. Cantor by a slip wrote the Sanskrit name of the product as tatstha. Sridhara's sutra is:

utsaryetsarye tatah kapata-sandhir bhaved idam karanam |
tasmin tisthati yasmat pratyutpannas tatas tatsthah ||

In this, tatstha has the sense 'that which remains in that place' (see the Trisatika). That which is born again in the place of the multiplicand is called pratyutpanna. From the prefixes prati and ut and the root pad (to go), pratyutpanna is made. Bhaskara in his Lilavati wrote this method of multiplication first of all, but did not tell

its name.

In the second method one makes optional parts of the multiplier, multiplies the multiplicand by each part, and by adding all takes out the product. This is the same method that is proved from the second proposition of the second book of geometry.

In the third method one divides the multiplier by such a number that the quotient comes whole. Then one multiplies the multiplicand by that number and multiplies the product by that quotient; that is the product. For example 12 x 135 = 3 x 4 x 135, or 10 x ... and so on.

The fourth, fifth and sixth methods are the remaining school forms: multiplying by the places of the multiplier one after another and adding (the ordinary modern column method); multiplying by aliqout parts; and the method of shifting as in tatstha, where the product is built in the same row as the multiplicand. Ganesa's seventh, the zarb-kothri or shabaka, is the lattice already described. Muslims in this region still use that name.

Division (pp. 74-78)

Division is bhajana or bhagahara. The dividend is bhajya, the divisor bhagahara or bhajak, the quotient labdhi, the remainder sesa. On dust the ganaka writes the divisor under the first enough places of the dividend, finds the first figure of the quotient by the pahare, multiplies, subtracts, and brings down the next digit — rubbing out as he goes. The working cannot live on paper without rewriting, which is why the Sanskrit books so often say only 'divide' and leave the key with the guru.

Bhaskara in the Lilavati, Sridhara in the Trisatika, and the later Aryabhata in the Mahasiddhanta all take the method as known. Checks of multiplication and division by nine — adding the place-digits until one digit remains — are written by the later Aryabhata for multiplication, division, square, square root, cube and cube root (see chapter 18 of my printed Mahasiddhanta). Al-Haytham wrote the rule of nine for addition between A.D. 965 and 1037. Iamblichus wrote that if of three neighbouring numbers the greatest is divisible by 3, then the repeated sum of the place-digits of their total ends at 6; as 64 + 65 + 66 = 195, 1+9+5=15, 1+5=6. That is the same thing the later Aryabhata wrote.

A better check is from 11: the difference of the sums of the odd and even place-digits, or the taking of each place-digit from the next, to the end; the last remainder is the remainder on dividing the number by 11. If a digit will not subtract from the next, add 11 and then subtract. Example: in 247896, 4-2=2, 7-2=5, 8-5=3, 9-3=6, 6-6=0, so 247896 is divisible by 11. In 283471 the remainder is 1. Lucas de Burgo wrote these checks for the four operations. Gerbert also wrote them; his pupil Otto III made him Pope as Sylvester II; he died in A.D. 1003. The method is not new; it is drawn from the old foundation (see verse 57 of the Chayadhikara in my edition of the Brahmasphuta-siddhanta).

Square and square root (pp. 78-82)

The Sanskrit name is varga-mula — the root of the square. A square is a figure of four equal sides; its root is one side. Aryabhata wrote one method of squaring and one of cubing, and for the square root and cube root he wrote the short working that is still famous everywhere. Brahmagupta, taking the method of squaring as well known, did not write the extraction of the square root, but he did write the cube-root rule that is current today. Sridhara in the Trisatika wrote both rules that are still current. Narayana Pandita in the Ganita-kaumudi followed Sridhara. Bhaskaracarya in the Lilavati wrote the same.

The working is this. From the unit place, mark the digits in pairs for a square root. Over the odd places a vertical stroke is set; over the even (non-square) places a horizontal stroke. Thus, to know the square root of 88209 one marks 8 82 09. In the left-hand group find the greatest square that does not exceed it; write its root as the first figure of the answer; subtract that square; bring down the next pair. Double the root so far

obtained; this is the trial divisor. Find the next root-digit so that divisor times that digit, plus the square of that digit in the next place, does not exceed the remainder. Continue to the end. The same geometry that makes a square out of a gnomon around a smaller square is the sense of the rule.

Sridhara in the Trisatika and Bhaskaracarya in the ksetra-vyavahara of the Lilavati also wrote how to take an approximate root of a non-square (avarga). Kamalakara, in A.D. 1658, in the Spastadhikara of his Tattvaviveka, proved clearly that the square root of a non-square is neither a whole number nor a fraction; one can only exhibit it as a line, and that line cannot be measured exactly. Many hold that Baudhayana did not know this numerical square-root working, and that he took the measure of the diagonal with a measuring-rod.

Cube and cube root (pp. 82-84)

Ghana is the cube; ghana-mula the cube root. Hemacandra's lexicon says ghana is used of a solid heap and of a deep sound. Aryabhata's rule marks digits in threes, subtracts the cube of the first figure, and uses three times the square of the root so far obtained as the principal divisor, correcting with the terms 3a^2 b + 3ab^2 + b^3. The work is heavier than square root and was less used in the village schools, but the jyotisis needed it.

Signs of plus and minus, which belong as much to arithmetic as to algebra, are noticed here so that the Hindi reader may see they are late in Europe. Diophantus, who died about A.D. 303, took a mark for minus, another for equality, and a mark for the unknown. Ahmes set a pair of legs walking forward for plus and walking back for minus. In Widman's arithmetic, printed at Leipzig in 1489, the signs + and - already appear. Stifel in the Arithmetica Integra, printed at Nuremberg in 1544, has the same signs. Later, in 1575, Xylander drew the upright of plus much longer than the cross-bar. Many think Stifel first made both signs.

Fractions (pp. 88-91)

A fraction is a bhinna, a broken number. Old Sanskrit astronomers thought the arithmetic of fractions hard; yet the adding, subtracting, multiplying and dividing of fractions was current in India long before Aryabhata, which is why he could take it as known in the Aryabhatiya.

The method of samaccheda (making denominators equal) shown by Brahmagupta and Sridhara makes it clear that those authors did not know the method of the least common multiple. Bhaskara wrote of reducing fractions by a cancelled divisor, but he too had not the working of the LCM. In the example beginning manikyastakam indranila-dasakam the rule he writes for the integer value shows that he did not know how to extract the LCM. Kamalakara in A.D. 1658 wrote a rule for the LCM in the Mahaprasnadhyaya of the Tattvaviveka (see my edition of the Siddhanta-tattvaviveka). In Europe Tartaglia first wrote the LCM rule in A.D. 1525.

Aryabhata and Brahmagupta did know the greatest common divisor. That is clear from their kuttakara mathematics. They knew addition, subtraction, multiplication, division, square, square root, cube and cube root of fractions. The same methods are still current in all Hindi arithmetics. If a whole number must be written as a fraction, Sridhara and Bhaskara say: put one as its denominator — chedanam acchedanasya rupam syat (Trisatika). Thus 3 is written 3/1.

A fraction of a fraction (bhaga-bhinna) is multiplied through. Continued fractions (anuvrtta-bhinna) appear when a ratio is expanded by repeated division, as in the kuttaka and in approximations to surds. Decimal fractions are the natural downward continuation of the ten-fold places. Jyotisa had long used sixty-fold places below the unit (kala, vikala). The modern decimal point is a European convenience (Stevin and those who followed him).

End of the close translation of printed pages 62-91 of the 1910 Benares edition. The next original stretch is printed pages 92-121 (decimal fractions, proportion, series, amicable numbers, interest, magic squares).

Close translation of the 1910 text, printed pages 92 to 121

These pages follow printed page 91 without a break: the Rhind papyrus and unit fractions; decimal fractions and how the mark was written in Europe; the signs of plus, minus, equality, multiplication and division; firm numbers (primes), the sieve, perfect numbers and amicable pairs; Fermat's theorems; and the filling of square-cells as yantra.

Ahmes and unit fractions (pp. 92-94, continued)

Many hold that the Egyptian book of Ahmes is from 3400 years before the Christian era. From this it is known that the people of Egypt knew mathematics five thousand years ago. On that book it is written: the road for knowing all the things of the dark chamber (Directions for knowing all dark things). This speech of a 'road' also comes in the Sanskrit books. Having described some matter, the acaryas at the end write 'iti dik' — that is, this is the road (direction, path) for knowing.

From this book it is proved that the people of Egypt were very raw in drawing out methods of mathematics. If 3x + 6, then x equals such-and-such. When r = 5, then x = 15. And in the value of x written above, when r = 6, then x = 18. Why Ahmes left one form and took another is not known. To make such fractions equal to the sum of two fractions with numerator 1 is not hard; yet Ahmes sometimes showed a fraction as the sum of several unit fractions, and sometimes otherwise.

There are also in that book some questions of one-unknown equations of bija, whose answers come by the ista-karma of Sanskrit arithmetic, with answers. For example: what is that number which, when its seventh part is joined to it, becomes 19? In taking out the answer Ahmes wrote x + x/7 = 19, and then x = 16 + 1/2 + 1/8. Ahmes called the unknown hau or heap. The answers of other questions were taken out by separate methods. In one place Ahmes made a number nine-fold in this way: first he doubled it, then doubled this double, then doubled this four-fold and joined the original to it.

In that book there are two parts. In the first the working is shown of bringing fractions to the form of two-thirds, then of bringing all fractions to the form of a sum of unit fractions. At the beginning of the second part there are questions of subtraction, division and ista-karma; further on some questions of a geometric series; and at the end two questions of an additive series. In this book it is also written that a debt multiplied by a debt makes the product a credit. A footnote: Diophantus, who died about A.D. 330 at the age of 84 and who was the famous algebraist of Greece, did not know the multiplication of negative numbers. That will be described in the algebra part.

Bringing a fraction to the sum of unit fractions is also found in Baudhayana's Sulba-sutra (see the Vedic section). The mind of the people of Egypt and Greece was on making some fraction equal to the sum of unit fractions; from this it is clearly known that they kept a measuring-rod (scale), divided it into parts equal to the denominator of a unit fraction, and took one part. But they gave no thought at all to the working changing when the denominators become different. Sometimes they kept 2/3 and 3/4 as they were, and sometimes they also set 2/3 = 1/2 + 1/6 and 3/4 = 1/2 + 1/4.

Decimal fractions (pp. 94-97)

Just as the astronomers of Babylonia used to reckon by taking a share of sixty and sixty, so the Sanskrit jyotisis also, in the reckoning of the planets, take a share of sixty and sixty, which they go on writing one under another below the unit. For brevity they do not write the denominator 60; they write only the numerator. Thus 2 amsas, 17 kalas, 21 vikalas, 24 prativikalas they will write one under another, and will understand 2 + 17/60 + 21/3600 + 24/216000.

In Hindustan there was no custom of reckoning after the whole number by taking a share of ten and ten. Under English rule this method has run here. In this region the men who started this method are Pandit

Mohanlal, Pandit Vamsidhar, Pandit Kunjbihari Lal and others. These men, by order of the government at Agra, first made for the reading of boys in the tahsili schools, by the English method, Ganita-prakasa, Ganita-nidana, Dasamalava-dipika and the rest; from then the arithmetic of decimals spread in these United Provinces. Those same men translated the English word decimal as dasamalava.

I have already written that although in Mr Peacock's opinion Napier is the inventor of the decimal, if one thinks truly, Regiomontanus is its inventor; because he first of all, leaving the Greek and Hindu method of dividing by sixties, for taking out the jya of trigonometry divided the radius into 1,000,000 parts. But the values of his jya, kotijya and the rest are in whole numbers; the fractional number was left.

In A.D. 1616, when Edward Wright printed the English translation of Napier's Mirifici Logarithmorum Canonis Descriptio, he had purified the decimals written in that book. I have already written that from A.D. 1619 to A.D. 1631 there is no mention of the decimal in any English book. Briggs wrote the decimal by giving a slanting line below; thus 2.57 he wrote as 2 with 57 underneath. In A.D. 1631 Oughtred wrote 0.56 as 0|56. A pupil of Stevin, Albert Girard, in A.D. 1629 wrote the decimal point in one place, and after this used the decimal everywhere in his algebra. De Morgan writes that after A.D. 1758 the decimal won everywhere; all people honoured it.

If you look and think, the decimal number is the little sister of the number that is written in the current ten-fold places; because in it all digits remain of tenth-part places. Therefore the elder sister, by force, having pushed out the foreign number of sixty-and-sixty shares, seated her little sister on her own left.

In Cantor's opinion, Pitiscus in A.D. 1612 first used the decimal in his trigonometrical table. Glaisher's opinion is that Rudolf also knew decimal arithmetic. In dividing, where the value of the divisor is some power of ten, they counted off to the left of the unit as many digits of the dividend as the power-number and there set the mark of a comma.

It is heard that Stevin, whose pupil spread the decimal point, made the decimal digits famous, like unit and ten, by the names tenth, hundredth, thousandth — first, second, third and so on. He wrote 7.628 as 7(0) 6(1) 2(2) 8(3). Kepler also kept a comma in the place of the point. Many say that Jobst Burgi, without seeing Stevin's method, wrote the decimal with a point in his table of sines as is done today, such as 0,32 and 3,2.

Not many days have passed since this decimal was born, yet, thinking it of much use, people at once, with honour, gave it a seat in their books. From the time of Bapudeva Sastri the use of the decimal has also begun in Sanskrit in this region.

Signs (pp. 97-103)

In the Sanskrit arithmetic books no signs of adding, subtracting, multiplying and dividing are found. In the books they write that number after its name. Thus viyojya 1520, viyojaka 520; gunya 135, gunaka 12; bhajya 1620, bhajak 40.

In algebra, to show a negative number, Bhaskara wrote: set a bindu above its head. Thus 3 with a dot is minus 3. And for a root he wrote the ka of karani; thus square-root of 3 is written ka-3. The signs of algebra will be written in the algebra part.

The Greek Diophantus, who died about A.D. 303, took a mark for minus, a mark for equality, and a mark for the unknown quantity. Other signs are not found in his book. Ahmes in his book set a shape like a man's two legs walking forward in the place of plus (dhana), and the shape of those legs walking back in the place of minus (rna). In some places he made three parallel arrow-heads for minus, and a mark like this for equality.

In Widman's arithmetic, printed at Leipzig in 1489, the signs of plus and minus are found as + and -. At that time in the plus-sign the upright line remained a little shorter than the slanting line. Stifel wrote a book on arithmetic named Arithmetica Integra, printed at Nuremberg in A.D. 1544; in it also the signs of plus and

minus are as in Widman's book. Later Xylander in A.D. 1575 drew the upright line of the plus-sign much longer than the slanting line.

In the thought of many people, Stifel is the first maker of both these signs. Many guess that these two signs are not the spoiled shape of the first letter of some word; both were simply made as signs. Many tell + as a kind of shape of the hand; but in my judgement this + is the spoiled shape of the first letter of et / plus, and - is the spoiled shape of the first letter of minus, as the multiplication-sign x is the spoiled shape of the first letter of the word which Stifel first wrote. Many hold that both are the spoiled shape of the first letters of more and less as then written.

Professor De Morgan's opinion is that the minus-sign - is a large form of the Hindus' rna-sign (the bindu). Afterwards, to show plus, this minus-sign was cut with an upright line and + was made. De Morgan wrote this in A.D. 1847 on p. 19 of his book related to mathematics. This is the same De Morgan who had Lala Ramchandra of Delhi's Maxima and Minima printed in Europe. In Europe he also started the custom of the postal money-order.

Many hold that + is a kind of shape of one man standing between two men and calling both to join him, one with each hand: the upright line is the spoiled shape of the head and back, and the slanting line the spoiled shape of both hands. Many say the Germans first made these plus-minus signs. However that may be, at first the use of both signs was among the banias. From the time of Vieta all people, for the ease of mathematics, began gladly to use them.

Robert Recorde in A.D. 1540, thinking that only in two parallel lines is there the most equality, made the sign of equality =. Recorde's algebra The Whetstone of Witte, made in A.D. 1557, is the first of all algebras in English. Johann Heinrich Rahn in A.D. 1659 made the sign of division; John Pell spread it in England in A.D. 1668.

Where Vieta had taken the large letters of the alphabet in bija, there, to save space, Harriot took the small letters, and made the signs > and < for greater and less. Ten years after Harriot's death his Artis Analyticae Praxis was printed in A.D. 1631. William Oughtred (1574-1660) made the multiplication-sign x and the sign of proportion :: . He had taken a sign for ratio which later, in the eighteenth century, Christian Wolff took for multiplication, and another for ratio. Many say that Oughtred and Harriot both brought into use in A.D. 1631 this multiplication x and this ratio-sign.

Pacioli and Tartaglia both used the minus-sign in division, ratio and proportion. Many hold that Descartes in A.D. 1637 took the point for multiplication. Leibniz in A.D. 1686 set a sign for multiplication and its reverse for division. Sometimes the Arab people set a slanting line between numerator and denominator, as 9-3 = 3; sometimes they also wrote 1/3 as 1/3.

Clairaut printed a book in A.D. 1760; in it also there is a sign for ratio. Multiplication is only a special working of adding; therefore a special form of the plus-sign + was taken as the multiplication-sign x. Division is only a special working of subtracting; therefore, giving one point above and below the minus-sign, the division-sign was such-and-such; its spoiled shape has become today's division-sign.

Recorde drew out the sign of equality =, and Xylander also used it in A.D. 1575, but until Newton its spread was very little. Newton in A.D. 1680 took for equality a sign like this, or its reverse; this appears to be the spoiled shape of the first letter of aequalis. Vieta set this equality-sign for difference. Oughtred made the proportion-sign rightly, but Wallis in A.D. 1686 spread its use everywhere. There was no need of making another sign of proportion from the sign of equality, because a:b = c:d has the same sense as a:b :: c:d.

Oughtred had first made signs for greater and less; later Harriot changed their shape a little to > and <, which after Barrow were started everywhere. The signs not-equal, not-greater and not-less have been started only a very few days. In A.D. 1591 Vieta spread parentheses, and in A.D. 1629 Girard spread the vinculum.

John Wallis, who in A.D. 1649 became professor of geometry at Oxford, made the sign of the infinite, that is infinity, as a lying-down 8. From the time of Muslim rule, signs of the anna, chatak, ser, tola, masa and so on have been made in Hindustan, which are well known everywhere. The remaining signs of algebra will be described in the algebra part.

Firm numbers (pp. 103-106)

Aryabhata in the kuttakara of his Aryabhatiya made no mention of a firm dividend-divisor. The arithmetic books of his pupil Prabhakara and others have not yet been found. From the saying of Paramesvara, commentator of the Aryabhatiya (see the Ganaka-tarangini), there is also a pupil of Aryabhata named Lalla, whom people for honour call Lallacarya.

Brahmagupta, in the kuttakadhyaya of his Brahmasphuta-siddhanta, having divided dividend and divisor by the greatest common measure, kept the name of those two the nicchedya bhajya-hara (dividend and divisor without a cut). After him no old arithmetic books of Balabhadra, Sripati, Sridhara, Padmanabha and the rest are found. It is certain that Sripati had an arithmetic (see the Ganaka-tarangini), but as his Siddhanta-sekhara is not found, so his arithmetic also has become rare. Sridhara's large pati is not found; in the small pati, the Trisatika (Patisara), there is no kuttaka-prakarana at all.

Bhaskara, in the kuttaka-vyavahara of his pati the Lilavati, having divided by the greatest common measure, kept the name of dividend and divisor the drdha bhajya-hara (firm dividend-divisor). Brahmagupta's nicchedya is itself Bhaskara's drdha. The jyotisis after Bhaskara have used drdha. Ganesa in A.D. 1520 wrote at the very beginning of his Grahalaghava 'drdha-guna-hara-lasat'.

In this way, in the old Sanskrit mathematical books, only the firm dividend-divisor is found. But what a firm number is called — this discussion in Sanskrit only Narayana Pandita has made in his Ganita-kaumudi; he wrote drdha as acchedya (uncuttable). Later Jagannatha Pandita, the pandit of the king of Jaipur (see the Ganaka-tarangini), wrote much theory of the firm in chapters 7-9 of his Rekha-ganita. That is a Sanskrit translation of an Arabic geometry. From the government side it has also been printed in the Bombay Sanskrit Series. Jagannatha Pandita finished this translation in A.D. 1718.

Euclid lived 300 years before the Christian era. He made this geometry by gathering the methods of his gurus and his own; therefore it is now very hard to find out whether the theories of firm numbers are Euclid's or his gurus'. It is known that, leaving the theories of firm numbers, all the rest were drawn out by the pupil-lines of Pythagoras.

After Euclid two famous men gave special attention to arithmetic. Eratosthenes, 275-194 years before the Christian era, wrote the method of knowing firm numbers. He wrote: if you wish to know how many firm numbers there are within 100, write down the odd numbers within 100 — 3, 5, 7, 9, ... 99. Then mark all the every-third numbers that are measured by 3; then mark all the every-fifth that are measured by 5. In the same way, from 7, 11 and so on, go on marking the numbers ahead that are measured by the unmarked numbers. When this is done, those that remain without a mark are the firm numbers within 100.

Just as among the Hindus people think omens from numbers — 'tin tikat mahavikat', 'car chandra kala' — so Aristotle and his followers also thought omens from numbers. This is known from Plato's book.

Perfect numbers and amicable numbers (pp. 106-109)

Euclid knew the method of summing the geometric series 1+2+4+8+... . Euclid also showed that if the sum up to some term of the geometric series above is a firm number, then multiplying that sum by the last number of the series makes a nidhi (perfect) number.

(That number which is equal to the sum of the divisors that measure it without remainder — Jagannatha Pandita kept its name nidhi in the definition of the seventh chapter of his Rekha-ganita. Thus the divisors that

measure 6 without remainder are 1, 2, 3, and 1+2+3=6; therefore we shall call 6 a nidhi.)

Thus 1+2=3 is firm; therefore multiplying it by the last number 2 makes the nidhi-number 6. In the same way 1+2+4=7 is firm; therefore multiplying it by the last number 4 of the series makes the nidhi-number 28. In this way, to this day these nidhi-numbers are known: 6, 28, 496, 8128, 33550336, 8589869056, 137438691328, 2305843008139952128. Further on, finding out whether the sum of the terms of the series is firm or not is great labour; therefore people have not found out further. I have written the method of knowing this nidhi-number in my Vastu-vicitra-praka; its proof comes from algebra.

Euclid knew the sum of the infinite series 1 + 1/2 + 1/4 + 1/8 + ... He took out the sum of this series for the area of a parabola (see my Cala-rasi-kalana).

The pandits of Pythagoras's school also drew out twin, yugma, yuga or paired numbers. If the sum of the divisors that measure the first number without remainder is the second number, and the sum of the divisors that measure the second number without remainder is the first number, then such two numbers are called yuga-sankhya (amicable numbers).

Example: the sum of the divisors that measure 220 without remainder = 1+2+4+5+10+11+20+22+44+55+110 = 284, the second number; and the sum of the divisors that measure 284 without remainder = 1+2+4+71+142 = 220, the first number. Therefore 220 and 284 are both yuga-sankhya.

The pandits of Pythagoras's school also knew much theory of firm numbers; there is no doubt of this, because from the definition of yuga-sankhya they were well acquainted, and then they drew out the method of knowing them. Their method is this: if p = 3·2^n − 1, q = 3·2^(n−1) − 1, r = 9·2^(2n−1) − 1 (n a whole and positive number) are firm, then a = 2^n p q and k = 2^n r will both be yuga-sankhya. For example if n = 2 then p = 11, q = 5, r = 71, all firm; therefore a = 220 and k = 284 became both yuga-sankhya. In this way, to this day three yuga-sankhya have been known: 220 and 284; 17296 and 18416; 9363584 and 9437056. Thabit also wrote a method of trisecting a given angle. In no Sanskrit mathematical book is there a discussion of yuga-sankhya. People of other countries have written much theory on firm numbers, of which my translation made into Sanskrit is also printed (see Karana-prakasa in the Chowkhamba Sanskrit Series).

That the firm numbers are endless, Euclid proved in the ninth chapter of his geometry. For knowing this firm, many have made many kinds of rules, but all become impure further on. One wrote that 2^n − 1 is a firm number; in this, when n = 4, 2^4 − 1 = 15, which is not firm.

Fermat's theorems (pp. 109-114)

In A.D. 1640 Fermat wrote in a letter a very beautiful theorem on firm numbers, which people now call Fermat's theorem: if p is a firm number and a and p are firm to each other, then a^(p−1) − 1 will be measured by p without remainder — that is, on dividing by p nothing at all will remain. This is Fermat's theorem (see Karana-prakasa).

Fermat also drew out a form 2^(2^n) + 1 for knowing firm numbers. Fermat could not show the proof of either of his two forms. The proof of the first form was later made by Euler (see Karana-prakasa). Until his death Fermat had full faith that he had drawn out the right form 2^(2^n) + 1 for knowing firm numbers, but he could not make its proof.

An American boy named Jera Coleb (Zacharias Dase / or rather the well-known later counter-example of Euler — Dvivedi names a lad) found that when in Fermat's form n = 5, then 2^(32) + 1 = 4294967297 = 641 × 6700417. Therefore from this form all firm-digits will not be born. How 641 came into the boy's mind he could not tell. Later Euler, showing this example, proved that Fermat's form is not right.

Fermat should be called the root-man of today's current theory of firm-digits; but who knows why Fermat used to make his methods famous among people and keep the proof hidden, by which other ganakas' time was wasted in thinking the proof for nothing. This same custom was also among the Sanskrit ganakas; but I have now lifted this custom away.

A French advocate, Bachet de Meziriac, in A.D. 1621 had Diophantus's mathematical book printed. Fermat got a copy of it; on the margins of its pages he wrote his methods like notes. After Fermat's death his son had that book of Diophantus printed in French with his father's notes. Fermat's other methods his son also had printed in Opera Varia and Wallis's Commercium Epistolicum in A.D. 1658.

Some questions of Fermat's notes: (1) Prove that in x^n + y^n = z^n this equation is impossible. On this Fermat wrote a note: I have proved it by a proof, but on the margin there is little room, therefore I have not written the proof. This same question later became among the ganakas the 'prize question' — that is, whoever answers it should get a prize. Euler, Lagrange, Dirichlet and Kummer were much harassed after this. Euler proved it impossible when n = 3. Lagrange proved it impossible when n = 4. Kummer showed impossibility in all values leaving some n; but Kummer's proof is not complete for every n.

Fermat also asked: (4) a firm number greater than 2 is the difference of the squares of two fixed numbers. He proved it thus: let the firm number be n; then according to the question (y − r)(y + r) = n. But n is firm, therefore y − r and y + r cannot be measured by it; therefore if you wish the equation right, y − r = 1 and y + r = n. (5) Prove that in y^2 + 2 = z^3 there is only one value of y, 5; and in y^2 + 4 = z^3 only two values of y, 2 and 11. Fermat challenged the English ganakas with these two questions. In this way Fermat asked many questions.

Euler in 1772, in the Memoires of Berlin, wrote for firm numbers the form y^2 + y + 41, which breaks at y = 40. 41, 43, 47, 53 ... forty firm numbers come right, but beyond that the form fails.

Series: Aryabhata's chiti (pp. 113-114)

Aryabhata, Brahmagupta and the rest knew the mathematics of an additive series. It appears that in Sanskrit the mathematics of a geometric series is from after Prthudaka. Aryabhata did not write the method of a geometric series, but he wrote the method of summing an additive series in his own verse. The sutra is:

> *ekottarady-upaciter gaccha-rdhaika-uttara-tri-samvargah |*
> *sad-bhaktah sa citi-ghanah saika-pada-ghno vimulo va ||*

If you make the shape of this chiti, then below set a triangle of 10 bricks, then in the second layer 6, in the third layer 3, and in the fourth layer 1; then on the base of this triangular pile there will be a suci (point) in which all the bricks = 1+3+6+10. In this, if the layer-term = n, then 1+3+6+10+...+ n(n+1)/2 = n(n+1)(n+2)/6. From this citi-ghana it appears that Aryabhata, who lived at Pataliputra, had seen such pointed piles built over the samadhi of the Buddhists; therefore he took out the count of the bricks stuck in them under the name citi-ghana.

Aryabhata also drew out the method of summing 1^2+2^2+3^2+...+n^2 and 1^3+2^3+3^3+...+n^3 under the names varga-citi-ghana and ghana-citi-ghana. Varga-citi-ghana = n(n+1)(2n+1)/6, and ghana-citi-ghana = [n(n+1)/2]^2 (see verse 22 of the Ganitapada). What Aryabhata called chiti, the people of other countries call triangular numbers.

Citi — series and piles (printed pages 115-118)

Aryabhata did not write clearly the method of the sum 1+2+3+...+n, but he wrote the method of summing an additive series in the Ganitapada of his Aryabhatiya:

> *istam vyekam dalitam sapurvam uttara-gunam samukha-madhyam |*
> *ista-gunitam ista-dhanam tv athavad y-antam padardha-hatam ||*

He has taken 'pada' (gaccha) and 'ista-dhana' as the sarva-dhana (the whole sum). Purva is the first number, which they call adi, mukha and the rest. From the sutra above, in that series whose adi = a, uttara (caya) = c, and pada = gaccha = n, the middle-dhana is (n−1)c/2 + a, and the ista-dhana = sarva-dhana = n/2 · (n−1)c + 2a. This same Bhaskara also wrote in his pati the Lilavati. In the sarva-dhana above, if a = 1, c = 1, n = p, then 1+2+3+...+p = p(p+1)/2. In this way we can say that Aryabhata knew the method of the sankalita beginning from one. He kept the name citi for this sankalita (see verse 22 of the Ganitapada).

Their sutra is:

ekottarady-upaciter gacchardhaika-uttara-tri-samvargah |
sad-bhaktah sa citi-ghanah saika-pada-ghno vimulo va ||

If you make the shape of this citi, then below set a triangle of 10 bricks, then in the second layer 6, in the third layer 3, and in the fourth layer 1; then on the base of this triangular pile there will be a suci (point) in which all the bricks = 1+3+6+10. In this, if the layer-term = n, then 1+3+6+10+...+ n(n+1)/2 = n(n+1)(n+2)/6. From this citi-ghana it appears that Aryabhata, who lived at Patna, had seen such pointed piles built over the samadhi of the Buddhists; therefore he took out the count of the bricks stuck in them under the name citi-ghana.

Aryabhata also drew out the method of summing 1^2+2^2+...+n^2 and 1^3+2^3+...+n^3 under the names varga-citi-ghana and ghana-citi-ghana. Varga-citi-ghana = n(n+1)(2n+1)/6, and ghana-citi-ghana = [n(n+1)/2]^2 (see verse 22 of the Ganitapada). What Aryabhata called citi, the people of other countries call triangular numbers. The method of their sum was known to Pythagoras, but the further methods of sankalitaikya and the rest perhaps were not known. Afterwards they became known to Diophantus, a pandit of his school.

Among the astronomers of the Arabs the first is al-Karaji, who gave the proofs of both methods: 1^2+2^2+...+n^2 = (1+2+...+n)·(2n+1)/3, and 1^3+2^3+...+n^3 = (1+2+...+n)^2. Afterwards Brahmagupta threw away the name citi and kept the names sankalita, sankalita-sankalita and the rest (see p. 188 of my printed Brahmasphuta-siddhanta). Then after him Sridhara, Bhaskara and the rest went on writing the methods. In Europe Pascal in A.D. 1653 wrote, under the name Traite du triangle arithmetique, the lineage of series of sankalita, sankalitaikya and the rest, from which Bhaskara had already written in his Lilavati (see p. 186 of my printed Brahmasphuta-siddhanta).

Bhaskara, for honour, calls Prthudaka Caturvedacarya. He was of Kannauj. He made an excellent commentary on Brahmagupta's Brahmasphuta-siddhanta. A broken old copy of that commentary is in the library of the India Office (see my introduction to the Brahmasphuta-siddhanta).

Interest proper (kalantara, vyaja) is not a separate chapter at printed pages 115-118 in the 1910 book; those pages are citi. Interest is treated as trairasika and as svamsanubandha at printed pages 124-126: if on a hundred in a month the interest be five; mixed sum; and year-by-year joining of a twentieth of the principal, 400 × (21/20)^4.

Magic squares and yantra (pp. 118-121)

Bhaskara wrote the working in his Lilavati (see p. 186 of my printed Brahmasphuta-siddhanta). Bhaskara, for honour, calls Prthudaka Caturvedacarya. He was of Kannauj. He made an excellent commentary on Brahmagupta's Brahmasphuta-siddhanta. A fragmentary old copy of that commentary is in the India Office Library (see my preface to the Brahmasphuta-siddhanta).

Yantra (magic squares). In a square of 3, 4, 5 ... cells, digits are filled with an increase of one in such a way that the sums of the numbers in the slanting, upright and corner cells are equal. Such a square-cycle the Sanskrit pandits call a yantra. Narayana Pandita kept its name bhadra. The sum is 15. Therefore this is called the fifteen-yantra.

On a year when, by the joining of the planets, Divali falls on a Sunday or a Tuesday, on that day in the great night (midnight) a tantrika, having purified himself, with a pot of sandal-paste and a pomegranate pen, writes in one breath many copies of this on a birch-leaf and keeps them by him. Hindus give that tantrika something and buy this yantra. They have it set in the middle of a silver yantra and bind it on the neck or the arm. The saying is that by wearing this there is no harm of Saturn, pretas, mahamari and the rest. From mathematics it is proved that by filling the nine cells with whole digits twenty (bisa) cannot be made; even so, until today village people remain in the toils of this and keep saying:

siddha hoya bisa | ka karai jagadisa ||

For kinds of work, kinds of bisa, tisa, chautisa and other yantras are filled. In all square-cells digits of this kind can be filled. Although this has no special connection with the science of mathematics — some work of a number-series falls — even so many people, thinking it a sport, have shown many methods of filling it. I too have written in the notes on Bhaskara's Lilavati the method of filling digits in all square-cycles; a separate Hindi translation of that has also been printed for all to understand. In it the method of filling digits also in the square-cycle of 4n+2 is written.

When the spread of these yantras began in Hindustan is not known exactly, but from behaviour it appears they have been current from a very old time. Narayana Pandita in his Ganita-kaumudi, which was made in A.D. 1356, wrote many methods of filling digits in these square-cycles and in other kinds of cycles. In it it is written that all these yantras were written for the son of (king) Sanibhadra. By filling digits in such square-cells many examples of adding are made, all of which have one and the same answer (see my yoga-cakra in Bhaskara's Lilavati).

In Europe first of all Moschopoulos, who died in Italy in A.D. 1470, thought much on these square-cycles. A book written by his hand is present in the National Library of Paris; among the books there its number is 2428. Later Euler in A.D. 1759 wrote many things on this in the Berlin Histoire de l'Academie. Those who need to know more on this subject should see Narayana Pandita's Ganita-kaumudi in Sanskrit; and if they wish to know the method of the pandits of Europe, they should see the European books on magic squares that Dvivedi cites at the foot of p. 121.

In tantra-sastra, having made kinds of cycles and having written words here and there in them, many yantras have been made. Among them a great mahima of the Sriyantra is written. Worshippers of Devi have it cut on sheets of copper, silver, gold or on crystal stone and worship it daily. They say that by worship the wish of the mind is fulfilled. The spread of this too is from a very old time. Hundreds of yantras of this kind are current in Hindustan. The saying is that he on whose neck a true bisa is bound is not touched by the blow of a sword. Many hold that this came from Hindustan.

Among the pandits of Pythagoras's school also the spread of yantras obtained. Tantra-vidya in China: those people had great faith in a five-cornered yantra. Those people had faith that by worship of this yantra the body remains without disease. Just as letters are written in the Sriyantra, so on the five corners of this the letters of a word (Dvivedi prints a foreign word on the plate) were set.

End of the close translation of printed pages 92-121 of the 1910 Benares edition. The next original stretch is printed pages 122-151 (combinatorics, logarithms, the special discussion, Vedic mathematics).

Close translation of the 1910 text, printed pages 122 to 151

These pages follow printed page 121: a carved yantra in Mysore and the life of Pythagoras; inverse working and the rule of three; interest as svamsanubandha; ista-karma and dvi-ista-karma; combinations from the meru of chandahsastra; logarithms (laghuriktha); a new foundation of number by assemblies of elements; and the opening of the Vedic measures from Baudhayana's Sulba.

A five-cornered yantra in Mysore, and Pythagoras (pp. 122-123)

In a village of the Hasan district of Mysore a yantra is cut on the stone of a gateway; in one of its cells there is also this pentagon. People have faith that by such a yantra remaining on the village gate no sickness spreads among the cattle. It is guessed that the pillar on which the yantra is cut is a thousand years old. (See Rice, Mysore and Coorg from the Inscriptions, 1909.)

Pythagoras was born at Samos 569 years before the Christian era. His mother and father were Tyrrhenian. He died at the age of 60. From this it is clear that he was in the time of Thales. There is much doubt in his life. So far as is known, he first studied with Pherecydes of Syros and afterwards with Anaximander. At the word of the guru, after studying, he went to Thebes or Memphis. He stayed there several years. Then, leaving Egypt, he travelled in Asia Minor, and then staying at Samos began to give lectures, but from this no fruit came. 529 years before the era, with his mother, he went to Sicily. He also took with him from Samos one worthy pupil. From there he went to Tarentum, but soon leaving it he went to Croton of the Dorian people in the south of Italy. Here he opened several schools in which the sons of great rich men began to study; therefore those schools in a few days became very famous, from which Pythagoras's great name was made. There the custom was that women should not come out and join the work, but the women broke that rule and used to go to see Pythagoras.

Pythagoras in old age married Theano, daughter of his kinsman Milo. Theano was very beautiful; she also wrote the life of her husband, but it is a matter of great sorrow that that was destroyed.

Whether Pythagoras came to Hindustan or not, there is much doubt in this. However that may be, the saying of many that Pythagoras came to Hindustan and studied mathematics with Aryabhata of Patna is a wholly false thing, because Pythagoras lived 1027 years before Aryabhata. It is possible that if Pythagoras came to Hindustan to study, he studied at Patna with some other pandit.

When the Turks in A.D. 1453 took Constantinople... [Dvivedi uses the fall of the city as a date-mark in the lives; the sentence runs on into the next topic of inverse working in the printed book.]

Inverse working (viloma-kriya) (pp. 123-124)

This is also called the vyasta method. Lalla's patiganita is not found, therefore we cannot say what he wrote. In this Brahmagupta made a speciality in verses 17-43 of the kuttakadhyaya of his Brahmasphuta-siddhanta:

> *gunaka-ccheda-cchedo gunako dhanam rnam rnam dhanam karyam |*
> *vargah padam padam krtir antyad viparitam ayatat ||*

In this, taking the root of a square and the square of a root — this much is special. The arithmetics of Balabhadra and Sripati are not found. Sridhara in his large pati may perhaps have written something special on this, but in his Trisatika (Patisara) there is no mention of this mathematics.

Last of all Bhaskaracarya in his Lilavati wrote the full method of viloma-kriya in every way, because where some part of the rasi itself has been joined to the rasi itself or taken from it, what to do in viloma-kriya — on this Brahmagupta wrote nothing at all.

The rule of three is the heart of pati (pp. 124-125)

In Sanskrit pati the most chief thing is trairasika. In Bhaskara's opinion trairasika itself is pati (arithmetic). He wrote in his pati the Lilavati: 'asti trairasikam pati'.

Knowing the value of a rasi by ista-karma; knowing the time of filling a cistern from pipes of water; dividing partnership-wealth; taking out the interest on a hundred; knowing a mixed sum and taking the interest apart; knowing the touch of golds of different rates melted and joined; taking one thing in exchange for another — for all these a method has been made from trairasika.

Svamsanubandha and svamsapavaha (interest) (pp. 125-126)

In Sanskrit pati there is one mathematics of svamsanubandha and svamsapavaha. Where some part of the rasi itself is to be joined to it, that is called svamsanubandha.

Example: at the rate of five rupees percent in a year, 400 rupees were given, and the condition was that at the end of every year the sum of interest and principal should be understood as principal; then at the end of four years what will the mixed sum be? Here every year a twentieth part of the principal itself is joined to the principal; therefore this became svamsanubandha or svabhaganubandha. By the method written in Sanskrit the answer is 400 x (21/20)^4.

Where some part of the rasi itself is to be taken away, that is called svamsapavaha. Joining is called anubandha and taking out apavaha; therefore from joining and taking out one's own parts or amsas this mathematics received the names svamsanubandha and svamsapavaha.

Ista-karma and dvi-ista-karma (pp. 126-128)

When, some amsas of the rasi itself having been joined or taken away, the value that remains is told, then the working of knowing the rasi from knowing that is called ista-karma. (In truth this question is of a one-unknown equation of bija.)

Example: what is that rasi whose third and fourth being taken out, 10 remains? In this suppose that rasi is the ista, 1; then by doing the work according to the question, 1 − (1/3 + 1/4) = 5/12 remains. Now multiplying the drsya 10 by the ista 1 and dividing by the remaining 5/12, the value of the rasi comes.

People of other countries say that in Sanskrit pati there is one mathematics 'dvi-ista-karma' for taking out the answers of many questions of algebra; but of all the Sanskrit patiganitas found to this day, in none is dvi-ista-karma. Perhaps it may be in the arithmetic of the Buddhists. I have written 'dvi-ista-karma' in the notes on Bhaskara's Lilavati. Bapudeva Sastri also wrote 'dvi-ista-karma' in his Hindi bijaganita and in the notes on Bhaskara's Lilavati. In the jyotisa-siddhantas the Sanskrit jyotisis, in taking out mahapata, certainly do the working by taking two istas, which is itself one kind of dvi-ista-karma (see the Patadhikara of the jyotisa-siddhanta).

By dvi-ista-karma the answer of many questions of a one-unknown equation of bija comes. From algebra it is proved that if f(x) = b is an equation, and one takes x = a and x = k, then f(a) = A and f(k) = K. By differencing, if only one power of x remains in f(x) = b, then the method of dvi-ista shown above itself becomes this, and the value of x comes exact. But when the square, cube and so on of x remain, the value of x will come by approximation.

In Sanskrit arithmetic methods are written for taking out the answers of many questions of algebra, whose description will be made in the algebra part. People of Europe guess the custom of that time from the questions of Bhaskara's Lilavati. From the question 'prapnoti cet sodasa-vatsara stri' one gentleman guessed that at that time a woman at sixteen years was understood exactly young. In Hindustan even now a woman of sixteen years is understood young. From 'mase satasya yadi panca kalantaram syat' it has been guessed that at that time five rupees on a hundred in a month — a very great interest — was taken; but in algebra in one place Bhaskara, in 'ekena satad attadhanat', wrote only one rupee interest on a hundred in a month. The only

meaning of saying this much is that in every time, when a famine falls, everyone's rate becomes sharp and slack. In such places a firm finding cannot be made from a guess.

Combinations: one-two... bheda (pp. 129-130)

In Hindustan the use of this mathematics is from a very old time. In chandahsastra the method by which this is taken out is called meru (mountain).

If the bhedas of 6 are to be taken out, then making a mountain of six stairs with one peak and an increase of one cell each, they will write one in the first cell and one in the cells of both edges of every stair. Then they go on setting in the lower cells the sums of the digits of the two neighbouring cells above — as in this mountain the last stair has 1, 6, 15, 20, 15, 6, 1. This is the meru-prastara of Pingala, which the people of Europe call Pascal's triangle. From it come the combinations of short and long syllables, and from those the binomial coefficients.

Permutations of digits and of things, with and without repetition, and the arrangements of n things taken so many at a time, are treated after Bhaskara and Narayana. The Sanskrit terms are sankalita, vikalpa, prastara, nasta and uddista. In Sanskrit pati, ksetra-vyavahara, kuttaka and anka-pasa are also written at much length. In the description of geometry, ksetra-vyavahara will be described; in the description of algebra, kuttaka and anka-pasa.

A closing praise of the maker of the digits (pp. 130-131)

The old Sanskrit pandits of Hindustan, thinking it a matter of pride, did not write their own lives. It is a thing of Jahangir's time: Bhattoji Diksita in his Siddhanta-kaumudi did not even write his name. There was some custom among the Sanskrit jyotisis of writing name and date; but at the time when there was no samvat and no Saka of anyone, in the heart of some Brahmarshi the root of mathematics and the places of the digits were born — therefore how should he write that date?

We people of the Veda call Brahma the first jyotisi of all, who is the maker of the Jyotisa-vedanga. He writes on the sixth day on everyone's forehead the account of good and bad works. Therefore that Mahabrahmarshi who made the digits, being Brahma, became the pitamaha of all; even if a sloka praise him, it cannot be finished. In the end this much is to be said: he whose praise Hindu, Muslim, Christian — all — make with one voice, that maker of the digits, who spread the fame of Hindustan through the whole world, is blessed.

A new imagination of number (pp. 131-132)

The opinion of the Naiyayikas is that from the joining of paramanus all things of the creation were born. From the joining of two paramanus a dvyanuka was made, from the joining of three a tryanuka. From the joining of few paramanus a small mustard-seed was made, and from the joining of many paramanus the great Meru (mountain) was made. Where the joining of however few paramanus remains, there that thing will be so much the smaller.

Therefore we can say that by the rule of the making of creation, from the meeting of clusters of the smallest number (the paramanu), these numbers 1, 2, 3 also will have been made. Thus in the infinite-term geometric series $1/2 + 1/4 + 1/8 + \ldots$ the last term will be the smallest, that is a paramanu. Therefore, writing the series reversed, the first term is a paramanu, the second two paramanus, the third four paramanus, and the sum of all the paramanus is 1 (from the mathematics of series).

Sine and cosine as series, and $a + b\sqrt{-1}$ — this they call a mixed number (complex number). From this thousands of new methods go on being made. Only in A.D. 1902 Mr E. T. Whittaker printed at Cambridge a very large book on this subject named A Course of Modern Analysis.

Logarithms: laghuriktha (pp. 132-139)

This mathematics was not in Hindustan. In no Sanskrit book is there mention of it. Bapudeva Sastri used it in his trigonometry under the name praghata-mapaka, but how it was made he did not mention at all. I understand that first of all in Sanskrit I myself wrote the method of knowing it in my Dirgha-vrtta-laksana. I myself kept its name laghuriktha. After dying, what wealth a father leaves, that in Sanskrit is called riktha. As when 1000 has died, what 3 remains, that you can call a small (laghu) riktha. The name matches the English name, in which those who know both English and Sanskrit have ease in remembering the name, and the name is also in one way meaningful; therefore I kept the name laghuriktha.

John Napier was Baron of Merchiston in Scotland. His birth was in A.D. 1550 and his death on 2 April A.D. 1617. It is a matter of great wonder that before the theorem by which logarithms are known had been known, Napier had already drawn out logarithms. The thought had come in the mind of Stifel and Stevin that numbers should be brought to the power-form of some one number, but those two did not find the trace of the logarithm. Harriot's algebra was published much after Napier's death, but in it also there is no mention of the logarithm. For many days it was not known that a logarithm is the exponent of some number. Afterwards first of all Euler...

Napier's own definition was kinematic. A point G runs along a line, and a point L runs along another; in the first instant both have one and the same speed, and in every instant the speed of L is equal, but the speed of G in any instant goes on lessening in relation to the remaining distance AG. Suppose that when G has run a distance Ag, L has run a distance Gh. Napier calls Gh the logarithm of AG. If AG = r, Ak = a and Gh = y, then from the calculus, Napier's logarithm of r = 10^7 log (a/r) in modern writing. From this it is clear that Napier's logarithm is not today's current Napier-logarithm (the natural logarithm). Napier had taken a = 10^7 as the radius (trijya) and r as the sine of some arc, and by the working above had taken out the logarithm of the sine. There is no doubt that Napier's logarithm is a kind of logarithm. If r = 10^7 then Napier's logarithm will be zero — that is, in Napier's opinion the logarithm of radius 10^7 is zero. Napier did not know in what base his logarithm is. By the method shown above Napier had made a table of the sines of all minutes within 90 degrees, with an increase of one minute, and of the logarithms of the tangent.

Many hold that the root-man of today's current logarithm is Burgi. His book was published much after Napier's book; therefore in Europe the honour of drawing out logarithms fell to Napier. Napier made a logarithm-table of the sines and cosines related to trigonometry, but Burgi wrote a table for the logarithms of all ordinary numbers. Burgi showed the first logarithms as 0, 1, 2, 3 and the second numbers as 1, 2, 4, 8. He also thought that if 10 be taken as base there would be great ease in the numbers of the second series. Many hold that afterwards the base 10 also occurred to Napier, but his life being finished he could do nothing.

Burgi's book of the geometric series was published at Prague; in it the logarithms of the numbers from 10^8 to ... are written. Burgi wrote the logarithms of numbers as red numbers and the numbers as black numbers.

Henry Briggs in Napier's time, in A.D. 1596, was professor of geometry at Gresham College, London, and afterwards in A.D. 1619 also became professor at Oxford. Seeing Napier's book he was astonished. Leaving all his work he went from London to Scotland to meet Napier, news of which had already reached Napier. Briggs was late on the road; from the delay Napier, becoming anxious, began to say to a friend: 'Ah, it appears that Briggs will not come.' At that same time the sound of a knock came at the door. On opening the door Briggs at once met Napier. For fifteen minutes both, from joy, remained looking at one another in silence. At the end Briggs said: 'My lord, I have come this far to see you and to give you thanks for this mathematics of the logarithm; blessed is your intellect which found the trace of this wonderful mathematics.' Then much talk passed between Briggs and Napier on this logarithm.

Briggs was born near Halifax in A.D. 1556 and had studied at St John's College, Cambridge. Mr Ward printed his life from A.D. 1640. He died on 26 January A.D. 1630. Having learned Napier's logarithm-table he thought that if the logarithms of all numbers were made in base 10 there would be great brevity in mathematics. Then, taking log 1 = 0 and log 10 = 1, he made the logarithms of 1-20,000 and of 90,000-100,000 to 14 decimal places. The logarithms of the numbers left in the middle a bookseller of Gouda in Holland, Adriaan Vlacq, completed.

In Europe the spread of logarithm-tables became very great. In A.D. 1875 the number of these tables was 5073, in which the number of decimal places is from 3 to 102. For use in behaviour a table to 7 decimal places is thought good. The table in which there are 102 decimal places, Per Gruson printed at New York in A.D. 1871 under the name Theoretic Tables. Later ganakas have gone on finding the impurities of these tables and purifying them. In Briggs's table, made in A.D. 1624 and printed in 1628, in which the logarithms of 1-100,000 are written to 10 decimal places, Lefort first found 171 impurities in the first seven decimal places, of which 48 were in the logarithms of 1-10,000. Vlacq slowly purified them all. Newton found errors in Vlacq's corrected table; Gardiner in 1742; Vega in 1797; Callet in 1855; and Sang in 1871 found 2 impurities. Glaisher, having examined well, established that in Bremiker's table printed in 1857, Sang's printed in 1860, Callet's printed in 1862 and Bremiker's printed in 1870, not one impurity was found. First of all John Speidell, in his book New Logarithmes, published the logarithms of sines, tangents and so on in base e.

A special discussion: number as an assembly of elements (pp. 140-150)

Here Dvivedi sets down a new account of what a whole number is, in the language of assemblies (samudaya) and elements (tattva). If four cows have come to be seen, we shall call 4 an assembly of the jati of one cow. In the same way an assembly of 4 paise is one anna. If in counting one anna you take one as the measure, we shall say that a rupee will be an assembly of sixteen of one anna. In this way all ordinary numbers are assemblies of their own one, and the tattva of every assembly is their one. If you call twenty-six gahi of mangoes one hundred, we shall say that in one hundred there are 26 tattvas and in one gahi 5 tattvas.

(2) From the counting of tattvas the knowledge of the mind becomes this: that this assembly is greater than, equal to, or smaller than another assembly. Therefore in the working of counting there is need of knowing the assembly, if that working further on comes to an end, or if there is firm knowledge of its coming to an end. Understand this: in the counting of some assembly its tattvas remain present separate from one another; it does not happen that some one disappears or that several join with one another. In the counting of the sound of heat (or of a clock-tick) the first sound disappears as soon as the second counting begins, but in the mind of the counter that first sound remains present.

(3) In the counting of the assembly of some thing, after one tattva a second comes, after the second a third, in order; therefore we can call some assembly an ordered assembly. We can call the first tattva of lower rank by the force of the second tattva, and the second of higher rank by the force of the first. (4) The fixed tattva of one assembly which equals a fixed tattva of another assembly is called a pair-tattva. If in one assembly there is no such tattva as can equal some tattva of the other assembly, we shall say that in both assemblies there is one such assembly in which, by the reckoning of the other, there are many more tattvas. In this way, from one, assembly, order and pair, these four...

(5) A bounded (paricchinna) ordered assembly is one in which there is a tattva of highest rank and a tattva of lowest rank. If Ma is taken as a part of the assembly Sa, then in Ma1 there will remain one tattva of highest rank and one of lowest rank, and in every part of Ma1, from Sa also having a part, there will remain one tattva of lowest rank and one of highest rank; therefore Ma1 itself will be a bounded assembly. (6) Two bounded ordered assemblies in which there are pair-tattvas in a complete way are called sajatiya (of one jati) — that is, if each tattva of one is the pair of each tattva of the other, such that no two tattvas of one are the pair of one

tattva of the other, and if Pa is of lower rank than Ka then Pam is also of lower rank than Kam, and if Pa is of higher rank than Ka then Pam is also of higher rank than Kam. From this it is proved that in two sajatiya bounded ordered assemblies there is one and the same ordinary number. If two ordered assemblies are each sajatiya of a third assembly, then those two will also be sajatiya of each other. The proof of this is very easy. From this it is proved that in sajatiya ordered assemblies some one fixed ordinary number remains stable.

(A) In the assembly in which there is only the one tattva A, the fixed ordinary number is one, which is published by the sign 1. From this it is proved that in all assemblies in which there is only one tattva the fixed number is 1. If in this a new tattva Ka is joined, then the new assembly (A, Ka) will be, where the rank of Ka is higher than the tattva A — that is, in rank A is small and Ka is great — then in the assembly (A, Ka) the fixed number will be 2. If in this again a third tattva Ga of higher rank than both is joined, then in the assembly (A, Ka, Ga) the fixed number will be 3. In this way in the assembly (A, Ka, Ga, ..., Ja) call the fixed number n, and join in this one tattva Jha of highest rank; then in the assembly (A, Ka, Ga, ..., Ja, Jha) the fixed number n' is clearly different from n.

By this construction, going forward, all assemblies become bounded ordered assemblies. From this it is proved that in one ordered assembly in which there is one tattva, as many assemblies as there will be from joining new tattvas one by one, all go on becoming bounded ordered assemblies. By the reverse working of this you can prove that in some bounded ordered assembly, taking away tattvas one by one, at the end there will be one such assembly in which only one tattva will remain. No bounded ordered assembly can be sajatiya of its own part. The proof of this is by anugama (induction) in this way... [Dvivedi runs the induction: if Ma is not sajatiya of any part of itself, then (Ma, ta) is not sajatiya of any part of itself; the two-element assembly is not sajatiya of its part; therefore 1, 2, 3, ... are all different from one another and empty things (independent).] The relation that is between blue colour and a blue pot — that same kind of relation is between an ordinary number and the related padartha.

Permutation of whole digits. If two bounded ordered assemblies A and Ka both have one smallest and one greatest tattva, then (A, Ka) is bounded. If A1, Ka1 are in order sajatiya of A, Ka, then we can set each tattva of A in the place of its pair-tattva of A1, and each tattva of Ka in the place of its pair-tattva of Ka1. By doing this there will be between (A, Ka) and (A1, Ka1) a pair-tattva (1, 1). Therefore the ordinary fixed number of (A, Ka) is the same as that of (A1, Ka1). In (A, Ka) and (Ka, A) the ordinary fixed number is one and the same; therefore a+k = k+a, which is well known from the working of adding. Think with attention: an ordinary number has no meaning at all until some assembly is shown by it. Therefore until two, three ... assemblies are joined, the sum of ordinary numbers also has no meaning.

(10) Take one bounded assembly whose number is k; if in the place of every tattva another bounded assembly whose number is a is set, then the number of the new assembly made in this way is called the product of k multiplied by a, and this is shown by a · k. From the arguments above it is well known that that new assembly will be bounded, and by setting sajatiya assemblies in its places its number will remain as it is. Therefore a · k will also be bounded. It is well known that a · k = a+a+a+... to k places. If in the new-made ordered assembly you change the order of the tattvas so that from all the assemblies whose number is a you take the first tattva and make one, the second tattva and make a second, and set those in order, then an assembly will be made whose number is a and in the place of each of whose tattvas the assembly whose number is k has been set. The number of this new assembly will be k · a, and this number is the same as the first — therefore a · k = k · a.

When the divisor will not go wholly into the dividend, great difficulty will have fallen. To wipe that away, the working of making shares of whole digits will have come into the mind; then the working of adding, subtracting, multiplying and dividing those shares will also have been thought. In the opinion of the Naiyayikas, where from the assembly the sum, difference and so on of whole digits have been shown above, a fractional number is not in the counting of number; but the ganakas began to do adding, subtracting and so on

in a fraction just as they do in a whole number; therefore they also began to call this by the name bhinna-sankhya. By k/n it is understood that one of some jati has been made into n equal shares and k such shares have been taken. Today's scientific people write a fractional number as (a, k), which the ganaka writes by this sign a/k.

I have sat down to write the history of mathematics in this small book; therefore I have given some description of the new opinion. Whoever has the wish to know all things should see The Theory of Functions of a Real Variable, made by Mr E. W. Hobson and printed in 1907. Now learned people, by the power of their intellect, may draw out whatever they wish; but that same root-man is blessed who made the shapes of these digits and the ten-fold places.

Vedic measures and mathematics (pp. 150-151, opening)

The definitions current today — 8 yava make an angula, 24 angula make a hasta, and so on — differ from the Vedic definitions. The maharsi Baudhayana writes the laksana of the angula in his Sulba-sutra: 'caturdasa-anavah | catus-trimsat-tilah prthu-samslistah ity aparam.' Anu means the small grains — saunva, kang, sarson and the rest. By setting fourteen anu-grains, or thirty-four tila, touching in length, one angula is made.

> *dasa-angulam ksudra-padam | dvadasa pradesah |*

Ten angula make one small foot, and twelve angula a pradesa. The gap that there is from spreading the thumb and the first finger, that they call pradesa.

Baudhayana writes in the 47th sutra of his Sulba-sutra: take a slanting line of twelve-angula measure; at one end of it make a side-line (koti) of seventeen angula less one tila. The diagonal (asrayya rajju) of the rectangle made from these two is twenty angula and twenty-seven tila. That is the trikarani (the side of a square-field three times the first square-field); the square-field made from it makes a (square-)field three times the first (square-)field. Its trtiya-karani (the square-field made by taking the third part of the side as side) is equal to a ninth part of the first square. Here the diagonal of the first square-field = √2 cubits, and from the sides 12 and 17−1/34 the karna comes, by Baudhayana's working, to 20 angula + 27 tila, which from the remainder is exact.

Baudhayana further, in sutras 61-62 of his Sulba-sutra, writes a speciality on the method written above: 'pramanam trtiyena vardhayet tac ca caturthenatma-catustrimsonena' — increase the measure by a third, and that by a fourth less its thirty-fourth. Increase a twelve-angula square; and that third by its own fourth less its own thirty-fourth, by one angula less a tila. Thus it becomes seventeen angula less a tila. This is Baudhayana's approximation to the square root of two, with which the next pages of the 1910 book continue.

End of the close translation of printed pages 122-151 of the 1910 Benares edition. The next original stretch is printed pages 152-181 (the rest of the Sulba, the closing dohas of the treatise, and the biographical appendix from al-Mansur onward).

Close translation of the 1910 text, printed pages 152 to 181

These pages finish Baudhayana's Sulba working, list the Sanskrit code-words for numbers, close the treatise with two dohas, and begin the short lives of the pandits named in the book, from the Abbasid caliph al-Mansur through Bhaskara and Muhammad.

Baudhayana's Sulba, continued (pp. 152-154)

Baudhayana writes in the 47th sutra of his Sulba-sutra:

pramanam dvadasangulam tiryak | tasya vikarani
tilona-saptadasangula parsva-mani |
evam-bhutasya dirgha-caturasrasya asrayya rajjur
vimsatir angulayah sapta-vimsati-tilas ca |
sa trikarani pramana-vimitam ksetram trigunam ksetram karoti |
trtiya-karany etena vyakhyata navamas tu bhumi-bhago bhavatiti |

Make one slanting line of twelve-angula measure; at one end of it a side-line (koti), perpendicular, of seventeen angula less one tila. The diagonal (asrayya rajju) of the long rectangle made from these two is twenty angula and twenty-seven tila. That is the trikarani (the side of a square-field three times the first square-field); the square-field made from it makes a (square-)field three times the first. Its trtiya-karani (the square made by taking the third part of the side as side) is equal to a ninth part of the first square.

Here the diagonal of the first square-field is the root of two cubits. And from the sides 12 and 17 less a tila the karna, by Baudhayana's working, comes to 20 angula + 27 tila, which from the remainder is exact.

Baudhayana further, in sutras 61-62, writes a speciality on the method above:

pramanam trtiyena vardhayet tac ca caturthenatma-catustrimsonena || 61 ||

Increase a twelve-angula square. Increase that third by its own fourth less its own thirty-fourth, by one angula less a tila. Thus it becomes seventeen angula less a tila. That is the visesa. From sutra 61 Baudhayana took $\sqrt{2} \approx 1 + 1/3 + 1/3\cdot4 - 1/3\cdot4\cdot34$. Setting 12 angula in the place of the side, the karna = $12\sqrt{2} \approx 12 + 4 + 1 - 1/34$ ... which is the well-known Sulba approximation to the square root of two.

Many hold that Baudhayana did not know the working of taking out a square root which I have written above; he measured the karna with a measuring-rod. If this is so, then Baudhayana's measuring-rod (scale) was very true and fine, from which even a tila was found. A special description of Vedic geometry will be made in the geometry-description part of this book.

Sanskrit words for the numbers (pp. 154-156)

1 = eka, rupa; names of the earth (bhu, bhumi, dharani); names of the moon (candra, sasi). 2 = dvi, yugma, dasra, the Asvins; names of Yama (yama, antaka); names of the hand (hasta, kara); names of the eye (netra, drk); paksa; yuga. 3 = trini, loka, Siva-netra; names of fire (agni, anala); Rama; guna; krama. 4 = catvari; names of the sea (abdhi, jaladhi); veda; sruti; astaka; krta; yuga. 5 = panca, prana; names of the arrow (bana, isu); indriya; names of the wind (vayu, pavana); bhuta; names of buddhi (dhi, medha). 6 = sat, rtu; names of the enemy (ari, ripu); rasa; anga; tarka. 7 = sapta; names of the mountain (acala, naga); svara; in the opinion of the ancients pavana, vayu; muni; names of the horse (asva, haya). (See the note on p. 25 of my printed Brhatsamhita with Bhattotpala's commentary.)

8 = asta; names of the elephant (danti, kari); names of snakes (naga, ahi); mangala; vasu. 9 = nava, nanda, anka, nidhi, graha; names of a hole (chidra, randhra, vivara, antara). In phalita they take the eighth place from chidra. 10 = dasa, pankti; names of the directions (dik, kastha). Somewhere they also take ten from Ravana-mukha, but this name is not found in mathematics. 11 = ekadasa; names of Mahadeva (rudra, siva).

12 = dvadasa; names of the sun (ravi, mitra); sometimes they take all 12 from the sun. 13 = trayodasa; names of the Visvedevas. 14 = caturdasa; names of Indra (sakra, suraraja); manu. 15 = pancadasa; day, tithi. 21 = ekavimsati; suci; svarga; prakrti. 22 = dvavimsati; akrti. Somewhere the greatest declination is also taken as 24. 25 = caturvimsati; tattva. 26 = sadvimsati; utkrti.

27 = saptavimsati; names of the naksatras (bha, tara). 28 = astavimsati. 29 = una-trimsat; in the Veda navavimsati. 30 = trimsat. 31 = ekatrimsat. 32 = dvatrimsat; names of the teeth (danta, dasana). 33 = trayastrimsat; names of the gods (deva, sura). 34-48 = the names that come in the well-known counting. 49 = una-pancasat; tana. 50-99 = the names that come in the well-known counting. 100 = sata; dasati, dasati are found somewhere in the Puranas. 360 = bhamsa, cakramsa; in the Jyotisa-vedanga bhamsa = 248 in one reading. 21600 = cakra-kala, bhagana-kala.

Closing of the first part (p. 156)

Ehi Angrezi-raj-bala saba desana ki riti |
samujhi bujhi lakhi manana kari bhaina para kara priti || 1 ||
Anka-ganita ki kachu katha likhi Sudhakara dhira |
tahi banci puravahu kasara nija budhi-bala lakhi hira || 2 ||

Thus: by the force of English rule the custom of every country — understand it, mark it, think it, and make love toward brothers. Sudhakara the firm has written some story of arithmetic; reading it, fill what is wanting, and looking by the force of your own intellect take the diamond.

Iti Sudhakara-dvivedi-krte Ganitetihase Patiganitetihasa-rupah prathama-bhagah samaptah.

Here ends the first part, in the form of a history of patiganita, in the History of Mathematics made by Sudhakara Dvivedi.

Lives of the famous pandits named in this book (printed pages 157-181)

Now, in brief, the lives of those famous pandits whose names have come in this book.

The Abbasid caliph al-Mansur (the second)

His first name was Abu Ja'far. On the death of his brother Abu al-Abbas he became the second caliph of Baghdad in A.D. 754. Being of the house of Banu Abbas or the Abbasids he is called Abbasid. His uncle, Abdallah son of Ali, fought him to become caliph himself, but was killed by his sardar Abu Muslim. He was a great lover of learning. He had many books translated into Arabic. In A.D. 775, going from Baghdad to Mecca, he died on the road at Bir Maimun. After death his body was brought to Mecca. Many hold he died at 63 and many at 68; he reigned 22 lunar years. He was very greedy; after his death there were 600,000,000 dirhams and 24,000,000 dinars present in his treasury.

Abu Ma'shar

Chief astronomer of the caliph al-Ma'mun at Baghdad. His full name is Ja'far ibn Muhammad ibn Umar Abu Ma'shar. He is called the prince of the later Arab astronomers. He was born at Balkh. The Sanskrit astronomical table which he translated into Arabic is named the Zij or Kitab al-Zij. A Latin translation of his book was printed at Venice in 1489. He died in A.D. 886.

Amarasimha

Counted among the nine jewels of King Vikrama. Varahamihira is also among those nine jewels; therefore Amarasimha was of Varahamihira's time (see Varahamihira).

Aristotle (Arastu)

Born at Stagira in Macedonia 384 years before the Christian era; died at the age of 62. He was a philosopher. He also thought about some questions of mechanics (yantra-vidya).

Al-Karaji (Al Karbi)

Of Baghdad; he was at the beginning of A.D. 1000. Among the algebras of the Arabs his algebra is counted the chief. He also wrote much on firm numbers.

Al-Qalasadi, Abu al-Hasan Ali ibn Muhammad

Very skilled in patiganita. He died in A.D. 1477, or as some say 1486.

Al-Nasawi, Abu al-Hasan Ali ibn Ahmad

In A.D. 1000 he was at Famanas in Khurasan. A good pandit of mathematics.

Al-Haytham (Alhossain)

Born A.D. 965, died 1037. One good mathematical pandit of the Arabs.

Asoka

Three hundred and twenty-three years before the Christian era the great Alexander, having conquered all people, held a darbar at Babylon in the hot season. The sardars said to Alexander: now give us jagirs from your kingdom. Alexander said: all people conquer Hindustan; that is a very great country; on conquering it I will give you jagirs there. The Greek sardars did not accept this. After the conquering of Hindustan the sardars had firmly resolved to keep its power in their own hands.

That same year, in the cold season, Alexander's death happened. The settlers of the Punjab, to obtain independence, killed the great sardars. The man who raised all the settlers was Chandragupta Maurya, of a poor house, who joined the men of the small jatis there and brought the whole Punjab under himself. Having

won the Punjab he marched on Dhanananda, king of Magadha, and in a short time pulled Dhanananda from the throne and sat on the royal seat at Patna and began to rule. At that time Magadha was counted the chief kingdom in Hindustan. Later this same Chandragupta became king of Hindustan as far as the ocean.

The victorious Seleucus, two years after Alexander's death, became satrap of Babylon and began to rule. People also call Seleucus king of Syria. Seleucus sent Megasthenes as his ambassador to Chandragupta's court. Seeing Chandragupta's power he wrote much in his book. Many also say that Seleucus gave his daughter in marriage to Chandragupta. Having ruled Hindustan twenty-four years, Chandragupta died. After him his son Bindusara, whom people also call Amitraghata, ruled Hindustan twenty-five years and died.

Two hundred and eighty years before the Christian era Seleucus died at seventy-eight; eight years after his death Bindusara's son Asoka sat on the throne at Patna and became king of all Hindustan. He was a great partisan of the Buddhist dharma. In all countries he had the Buddhist teaching cut on stone pillars and ordered the prajas to walk according to it. He ruled Hindustan thirty-eight years. The praja-people were very glad to see him; therefore they called him Priyadarsin. Many stories of him are written in the Buddhist books. Vincent A. Smith, M.R.A.S., also printed an English book named Asoka at Oxford, at the Clarendon Press, in 1901.

Archimedes

Born at Syracuse 287 years before the Christian era; killed at the age of 75 by the hand of a Roman soldier. He studied at the University of Alexandria with Conon. His power in yantra-vidya was unmatched. After studying he spent his life remaining in Sicily. Whenever some difficulty fell to King Hiero of that place in some matter, that work used to be done by his advice.

Aryabhata

Of Patna. At the age of twenty-three he wrote an astronomical siddhanta-book which people call the Laghu-Aryabhatiya siddhanta. That book was written in A.D. 499. (See the Ganaka-tarangini.)

The later Aryabhata

His made book is the Mahasiddhanta, which I have printed with my own commentary in the Benares Sanskrit Series. In Balambhatta Diksita's opinion his time is A.D. 953 (see my subject-index in the Mahasiddhanta). In this same man's Mahasiddhanta a very easy method is written for checking multiplication, division, square, square root, cube and cube root. From this same method today's method of taking out 9 has come.

Albert Girard

Lived A.D. 1590-1634. He has a book on the theory of equations.

Albrecht Durer

Born at Nuremberg in 1471; died there in 1528. A great painter. He is the first author of the method of perspective.

Eratosthenes

Born at Cyrene in Africa 276 years before the Christian era; died at Alexandria at 82. Very skilled in geometry. Knowledge of the firm numbers first came to him; therefore people call him the Sieve.

William Oughtred

Born at Eton on 5 March 1574; died at Albury in Surrey on 30 June 1660. He has books on pati and on trigonometry.

Kamalakara

Son of Nrsimha, a Maharashtrian Brahmin. His ancestors lived at Gola village on the north bank of the Godavari, but his father came to live at Benares, and he too lived at Benares with him. He studied with his elder brother Divakara. In A.D. 1658 he made the Siddhanta-tattvaviveka. In the year he made that book, Newton of Europe was sixteen years old. (See the Ganaka-tarangini.)

Kummer

Born 1810, died 1893. Professor at the University of Berlin. He wrote much on mixed numbers $(a + b\sqrt{-1})$ and on firm numbers.

Krsna Daivajna

Chief pandit of the Emperor Jahangir. His father's name was Ballala and his mother's name was Gopi. His Navankura commentary on Bhaskara's Bijaganita is famous. That commentary was made about A.D. 1600. For other matters see the Ganaka-tarangini.

Johann Kepler

Born near Stuttgart in Wurttemberg on 27 December 1571; died at Regensburg in 1630. He was a siddhanti and the helper of Tycho Brahe. He laboured to bring logarithms into use. In 1609 he drew out three principles on the motion of the planets, on the basis of which the science of siddhanta made great progress. He was much grieved by the death of his wife. A second learned wife also went mad in a few days. He was a great believer in phalita-jyotisa. It was for phalita that, having observed, he drew out the three principles of the planetary orbits.

Cardan (Girolamo Cardano)

Born at Pavia on 24 September 1501; died at Rome on 21 September 1576. Teacher of mathematics at Bologna and Padua. In 1539 he went to Tartaglia's house in Milan and with great entreaty swore he would tell no one the method of solving the cubic; having learned it he broke his word and in 1545 printed it as his own.

Clairaut, Alexis Claude

Born at Paris in 1713; died there in 1765. Powerful in the thought of curves.

Ganesa Daivajna

Born A.D. 1507, son of Kesava of Nandigrama. His Buddhivilasini commentary on Bhaskara's Lilavati is famous. (See the Ganaka-tarangini.)

Gerbert (Pope Sylvester II)

Died A.D. 1003 at the age of about 59. In A.D. 999 he was at Rome. Skilled in music-sastra and in the astronomical siddhanta. Afterwards he became Pope. Hearing his great name, Hugh Capet had his own son Robert called from Reims. There Robert later sat in the name of the second Otto (Otto II) as king of France.

Gerhardt

In A.D. 1848, at Hanover, in the library of Leibniz, some papers of Leibniz were found on which was written Observationes Philosophicae in itinere Anglicano sub initium Anni 1673. Leibniz had first gone to London; he returned at that same time.

Gellibrand, Henry

A.D. 1597-1637. Professor of astronomy at Gresham College.

Govindacarya

Lived at the door of Benares. After the English came, he began to live at Vindhyachal near Mirzapur, and died there about A.D. 1830. His father's name was Govinda. In his time he was a famous man in astronomy and in tantra-sastra. (See the Ganaka-tarangini.)

Gautama Buddha

The capital of the small kingdom of the Sakyas was Kapilavastu. Its king was named Suddhodana. They were of the Gautama line. 557 years before the era a son was born to Suddhodana from Queen Mahamaya; the name the family set was Siddhartha. This same Siddhartha afterwards, becoming siddha by yoga, became famous by the name Buddha. From being born in the Gautama line people also call him Gautama Buddha. His siddhanta was that whether high jati or low jati, all can obtain the pada of nirvana by setting unmoving dhyana on Paramatma. And the true dharma of man is this: keep daya on all jivas; do not at any time do himsa of any jiva — ahimsa paramo dharmah. After becoming siddha his first discourse was at Benares, Sarnath. At eighty years of age, when he had come eighty miles east of his birthplace to give a discourse, then without the pain of disease he obtained the pada of nirvana. That place is near Kushinagara.

Glaisher

Born A.D. 1848. Professor of mathematics at Trinity College.

Charles Martel

Son of Pepin of Heristal, king of France. After the father's death Chilperic II, king of the Franks, drove him from his place. Later, in October A.D. 732, he conquered his enemies and took his kingdom again. He died in A.D. 741.

Jagannatha Pandita

Chief pandit of Raja Jaisingh of Jaipur. By Jaisingh's order he made geometry and the astronomical siddhanta (the Samrat) into Sanskrit from Arabic. He was in A.D. 1718. (See the Ganaka-tarangini.)

Jayaram Jyotisi

Son of Babua Jyotisi, a Maharashtrian Brahmin living at Benares. Babua Jyotisi himself gave the muhurta to Wazir Ali for fighting the English (see my Ganaka-tarangini). Jayaramji was skilled in jyotisa, vyakarana, nyaya and many vidyas. He was present in the time of Durgasankara Pathaka. He died in A.D. 1795.

Johann Christian von Wolff

Born at Breslau in 1679; died at Halle in 1754. Professor of mathematics at Halle and Marburg. He made books fit for reading in the schools.

John Pell

Born in Sussex on 1 March 1610; died in London on 10 December 1685. He translated Rahn's algebra.

Tartaglia (Niccolo Fontana)

His real name was Niccolo Fontana. Born at Brescia in A.D. 1500; died at Venice on 14 December 1559. In A.D. 1512, when the French people began to loot the city, the people of the city fled and hid in the church; there all were killed by the hands of the soldiers. His palate and jaw were also cut by someone's sword. The soldiers, thinking him dead, left him; but the mother saw that the boy was alive, so with great love she lifted him and brought him. Being poor the poor woman could not do medicine. She used to have a dog lick that wound. By the grace of Paramatma the wound became well, but from the palate being cut he could not speak well; therefore people began to call him Niccolo. The mother made some arrangement of writing and reading, but they were so poor that they could not even buy paper for writing. Later, about A.D. 1535, he became professor of mathematics at Venice and in a few days became very famous. The method of solving the cubic

equation is his drawn-out method, which people call Cardan's method.

Ptolemy (Claudius Ptolemaeus)

Born at Ptolemais in A.D. 87. Died at Alexandria. Among the Greek siddhantis a famous man.

Diophantus of Alexandria

Born about 275 years before the Christian era. Very skilled in algebra. He thought much on kuttaka and varga-prakrti. His methods will be written in the algebra part. He died at the age of 84.

Descartes, Rene

Born at La Haye in Touraine in A.D. 1596; died at Stockholm in A.D. 1650. He drew out analytical geometry. He drew out many new things in algebra.

De Morgan, Augustus

Born in June A.D. 1806 in the Madura district of the Madras Presidency, where there is the great temple of Minaksi Devi; died in Europe on 18 March 1871. In A.D. 1828 he became chief teacher of mathematics at London University. Very skilled in algebra. He had Lala Ramchandra of the Punjab's Maxima and Minima printed in London. In his time he was an unmatched pandit of mathematics. (For Lala Ramchandra see my Cala-rasi-kalana.)

Dirichlet, Gustav Lejeune

Born at Duren in 1805; died at Gottingen in 1859. He wrote much on firm numbers. After Gauss he was professor of mathematics at Gottingen.

Thabit ibn Qurra

Born at Harran in Mesopotamia in A.D. 836; died at Baghdad in 901. A great siddhanti and reckoner. He also made an Arabic translation of Srikarna's work. He also wrote much on firm numbers.

Tulasidasa

A Suryavamsin Brahmin living at Rajapur village in the Banda district. He died at Asighat in Benares on the seventh of the bright half of Savan, Samvat 1680 (A.D. 1623). His Bhagavata-Ramayana is famous house by house. He made twelve chief books. (See the Ramacaritamanasa printed by the members of the Kasi Nagari Pracharini Sabha at the Indian Press, Allahabad.)

Theodorus of Cyrene

Lived 470 years before the era. A pandit of mathematics in Plato's school. His writing on surds is found.

Dinadayal Ghanakshara

First he lived at Dehlavinayaka of Pancakosi in Benares; later he began to live at Kasi Sakhivinayaka. He was a very good poet of the Hindi language. Babu Harishchandra's father Gopalchandra, Lokanatha Chaudhri, my father Pandit Krpaladatta — all were his pupils. He made many books; among them Anuragabhaga and Anyaucityaharstra are very famous. He died at Benares about A.D. 1877.

Durgasankara Pathaka

An Audicya Brahmin living at Benares; he studied with his brothers Sivalal and Laksmi pati. In his time an unmatched jyotisi. Maharaja Ranjit Singh of Lahore sat on the throne by his muhurta. He made the janmapatri of Nau Nihal Singh of Lahore.

Narayana Pandita

Son of Nrsimha Pandita. In A.D. 1356 he made the Ganita-kaumudi. In it there is discussion of magic squares. No discussion of firm numbers. He wrote much on combinations of numbers. In the beginning of the Ganita-kaumudi it is written that these yantras were written for the son of (king) Sanibhadra.

Panini

The grammarian of the Astadhyayi. Patanjali's time is after A.D. 150. Panini's country is Salatura; Patanjali's is Gonanda-desa (Gonda). They call him Gonika-putra and Gonandiya.

Peacock, George

He made a history of arithmetic.

Pitiscus, Bartholomaeus

He has a book on trigonometry.

Paulisa

He was before Aryabhata. A good reckoner. A copy of a commentary is in the Benares Sanskrit College. He is an adi-purusa of the siddhanta.

Burgi, Joost

He wrote on logarithms.

Bapudeva Sastri

A Maharashtrian of the Pavana branch (see the Ganaka-tarangini). He introduced the English slate at the Benares Sanskrit College and used logarithms in Sanskrit under the name praghata-mapaka.

Bachet de Meziriac

Born ... died A.D. 1638. Famous for printing Diophantus.

Brahmagupta

See the Ganaka-tarangini. Born A.D. 598. Author of the Brahmasphuta-siddhanta (Saka 550) and the Khandakhadyaka.

Balabhadra

After Brahmagupta. His arithmetic is not found.

Bhattoji Diksita

See Bhanu Diksita.

Bhattotpala

In the mangalacarana of his commentary on the Brhatsamhita he wrote his name Utpala from 'dvijavara-strikam karoty utpalah'; even so people with honour call him Bhattotpala. His commentary is found on all of Varahamihira's books except the Pancasiddhantika. At the end of the Brhajjataka-tika it is written that I made this commentary in Saka 888, that is A.D. 966. Therefore he was in A.D. 966. (See the Ganaka-tarangini.)

Bhanu Diksita

Son of Bhattoji Diksita, maker of the (vyakarana) Siddhanta-kaumudi. At the word of Srikirtisimha Deva of the Baghel line, king of Madhidhara-desa, he printed the commentary Vyakhya-sudha on the Amarakosa. Later he became a sannyasi; at that time his name was set Ramashrama. In the house of Sri Ganesadatta

Tripathi, Sanskrit pandit of the Benares Collegiate School, there is a book of the Siddhanta-kaumudi written in Samvat 1738 (A.D. 1681); from this it appears that Bhattoji Diksita was in the time of the Emperor Jahangir. Bhanu Diksita's son was Hari Diksita. His pupil is Nagesa, who made the Laghu-sabdendu-sekhara. Many say that Bhattoji Diksita's birth was in Saka 1500, that is A.D. 1578. His house was in Bangali Tola, Benares.

Bhaskara (Bhaskaracarya)

Born in A.D. 1114 at Bijapur village in the tarai of the Sahya mountain in the south Karnataka country. His father's name was Mahesvara. He studied with his father himself. At the age of 36 he made the Siddhanta-siromani. He was the unmatched mathematician of his time. In A.D. 1183 he made the Karana-kutuhala. From this it is clear that he died at more than 69 years of age. If you wish to know other matters, see my Ganaka-tarangini.

Muhammad

Born at Mecca on Monday 20 April A.D. 572. He received the teaching of dharma from Ismail, son of Abraham. [The notice continues on the next original pages.]

End of the close translation of printed pages 152-181 of the 1910 Benares edition. The next original stretch is printed pages 182-207 (the rest of the lives, the royal chronology, and the Hindi word-index).

Close translation of the 1910 text, printed pages 182 to 207

These pages finish the lives, then give the sabdanukramanika (word-index) of the Hindi book. The 1910 volume ends at printed page 207.

Mahidhara

He made a bhasya on the Veda named Veda-dipa.

Madhavacarya

He is the son of Mayana. His younger brother is Sayana, who made the Vedartha-prakasa bhasya on the Veda. Madhavacarya wrote in his Sarva-darsana-sangraha:

srimat-sayana-dugdhabdhi-kaustubhena mahojasa |
sarva-vedyesu vedanam vyakhyatr-tve niyujyatam ||

From this it is clear that his younger brother Sayana himself made the Veda-bhasya. Madhavacarya was the chief minister of King Bukka of Ijanagara (Vijayanagara). At his word Sayana made the Vyakarana-Madhaviya-dhatu-vrtti. In A.D. 1896 the chief teacher of grammar at the Benares Sanskrit College, Mahamahopadhyaya Damodara Sastri, purified this Madhaviya-dhatu-vrtti and printed it in the Pandita-patra. From King Bukka's copper-plate it is clear that he was present in A.D. 1391.

Munisvara

His ancestors lived at Dadhi village of Ela-desa on the bank of the Payosni river. Later his grandfather Trimala came away to Benares. His father's name was Ranganatha. He was born at Benares in A.D. 1603. His Nisrstartha-duti commentary on Bhaskara's Lilavati and the Marici commentary on the Siddhanta-siromani are very famous. The first half of Marici was made in A.D. 1635 and the second half in A.D. 1638. He made the Siddhanta-sarvabhauma in A.D. 1646. For other matters see the Ganaka-tarangini.

Iamblichus

He was about A.D. 325. There are essays of his on all the mathematics.

Euclid

Three hundred years before the Christian era, in the reign of Ptolemy Soter, he was at Alexandria. He collected the thirteen chapters of geometry.

Euler, Leonhard

Born at Basel in A.D. 1707; died at St Petersburg in A.D. 1783. In his time an unmatched pandit of siddhanta and of mathematics.

Rudolff, Christoff

Born about A.D. 1525. An algebraist of Germany.

Robert Recorde

Born at Tenby, Wales, in A.D. 1510; died in a London jail in A.D. 1558. Professor of mathematics at Oxford.

Ramses

In Egypt they call the Suryavamsi Ramses. The third Ramses Herodotus calls Rhampsinitus. About 1200 years before the Christian era he ruled Egypt about twenty-five years. In 1886 his mummy was found. Much of his time passed only in war.

Regiomontanus (Johannes Muller)

Born at Konigsberg on 6 June 1436; died at Rome on 6 July 1476. Very skilled in mathematics and siddhanta. He translated many Greek books. First of all, the trigonometry he made was fixed for reading in the schools.

Lalla

In the opinion of Paramesvara, commentator of the Bhatadipika on the Aryabhatiya, he was a pupil of the first Aryabhata. His father's name was Bhatta Trivikrama and his grandfather's name was Samba. His made book is the Sisya-dhi-vrddhida. Bhaskaracarya in his Siddhanta-siromani made much khandana of this same book. (See the Ganaka-tarangini.)

Lagrange, Joseph Louis, Comte

Born at Turin on 25 January 1736; died at Paris on 10 April 1813. In his time an unmatched pandit of mathematics. He found the trace of many new things in mathematics, on the basis of which the science of mathematics has made great growth. He was also very skilled in the astronomical siddhanta.

Vatesvara

He made one astronomical siddhanta; in it there is khandana of many of Brahmagupta's sayings. In that same siddhanta it is written that in Brahma's life only eight and a half years have yet passed (ka-janmano 'stau sad-ardhah samah prayuh). He was after Brahmagupta.

Varahamihira

Son of Adityadasa. In Bhattotpala's opinion he was of Magadha. From being asrita at the house of King Vikrama he later began to live at Ujjayini itself. In A.D. 505 he made the Pancasiddhantika. His made books Brhatsamhita, Brhajjataka, Laghusamhita, Laghujataka and Yoga-yatra are famous. (See the Ganaka-tarangini.)

Wallis, John

Born at Oxford in A.D. 1616; died at Oxford in A.D. 1703. Professor of geometry at Oxford; he published many books of mathematics.

Widman, Johannes, of Eger

He was in A.D. 1489. At Leipzig he used to give lectures on algebra. First of all in the German language he himself made algebra. He also has books on geometry and on patiganita.

Vieta (Francois Viete)

His full name is Francois Viete, seigneur de la Bigotiere. Born at Fontenay-le-Comte in A.D. 1540. He drew out many new methods in algebra and is counted among the root-men of modern European algebra.

Vaisnavadasa Baba

A Marwari Vaisya living at Delhi. In Samvat 1890 he left the house and came away to Benares, and having studied vyakarana, vedanta, nyaya, mimamsa well with Pandit Sri Bhairava Misra, who made the Bhairavi commentary on the Sabdendu-sekhara, becoming a Vedanti sadhu, he began to live in the akhara of the Vedanti Tulsidas near Chauka Ghat in Benares. From his teaching hundreds of pupils became good pandits. He died on the tenth of the dark half of Jeth, Samvat 1936, A.D. 1879. At the time of death his age appears to have been eighty years. One day before dying I went with my father (Pandit Krpaladatta) to meet him. He was not at all ill, but he said to my father that tonight I shall leave my body; on this I said 'Still you will live ten years.' Hearing my word he scolded me that you are still a boy, you cannot know this thing. After

we people came home, that same day at ten o'clock at night he died. I also used to learn some yoga-kriya from him.

Sankar Balakrsna Diksita

He wrote in the Marathi language The Indian Calendar in A.D. 1896. At Poona he became a very famous man. He wrote much.

Raja Sivaprasad Sitare Hind

His birth was on the second of the bright half of Magh, Samvat 1880 (A.D. 1823), and his death on 23 May A.D. 1895 at Benares. He is of the Jaina Gokharu line. In spreading Hindi he should be called an adi-purusa. His grandfather's name was Raja Alacand and his father's name Govindacand. At the word of Pandit Mathuranatha Malaviya (who was librarian of the Benares Sanskrit College) his mother used to do daily puja of Siva (Mahadeva) for a child. Therefore on the boy being born the name Sivaprasad was set. He made many books for the rise of Hindi. For other matters see the Hindi-kovida-ratnamala.

Sridhara

Many people call him Bhatta Sridhara. He made pati, bija, varsa-paddhati, Trisatika (Patisara) and other books, but not all are found. I have printed the Trisatika with proof at the Chandra Prabha Press. In my opinion his time is A.D. 991. (See the Ganaka-tarangini.)

Sripati

Many people also call him Sripati Bhatta. His made pati, bija and Siddhanta-sekhara are not found. In jataka the Sripati-paddhati, and in muhurta the Ratnavali, Ratnasara and Ratnamala, these books are found. One karana-book of his named Dhi-koti is also found, which was made in A.D. 1039. From this it appears that he was in A.D. 1039. (See the Ganaka-tarangini.)

Sriharsa

In the Jain Rajasekhara's Prabandha-kosa, which was made in A.D. 1348, it is written that Sriharsa son of Hira was born at Benares. By the order of Jayantacandra, son of Govindacandra king of Benares, he made the Naisadha-carita. Sriharsa's mother's name was Mamahadevi, which is clear from the sloka at the end of the pratisarga of the Naisadha-carita. The chief teacher of the Benares Sanskrit College, the late Pandit... [the printed page is broken here] ... that Kanaujiya Hira Misra was Dara-Sriharsa who was the chief pandit of the king of Kanauj. From this it appears that Hira Misra in Sanskrit was called Hora-misra and Dara-Sriharsa. Jayantacandra ruled at Benares in A.D. 1168-1197; therefore that same is also Sriharsa's time. Whoever has the wish to know more should see Dr Georg Buhler's paper On the History of the Naisadha-carita, read on 9 November 1871 and printed in 1875. Bhaskaracarya's birth is in A.D. 1114. He made the Karana-kutuhala in A.D. 1183. From this it is clear that Bhaskara and Sriharsa are of one and the same time. On the subject of Sriharsa, Mahamahopadhyaya Sri Sivadatta, chief pandit of the Oriental College at Lahore, has written much in the introduction to the Naisadhiya-carita printed with the Narayana commentary.

Alexander the Great

He was born 356 years before the Christian era. Muhammad in the Quran calls Alexander Zu'l-qarnain (the two-horned). Malik Muhammad also in the Padmavat writes him Zu'l-qarnain. On his coin also a two-horn is made. 331 years before the era he conquered Dara, king of Persia. 327 years before the era he made an invasion on Hindustan. Without stop he came across the Sindhu river, but after crossing he was stopped from going further by the king of the Punjab. The Greek people wrote the name of the king of the Punjab as Porus. He died at the age of 33.

Stifel, Michael

Born at Esslingen in A.D. 1486 or 1487; died at Jena in A.D. 1567. He became very famous from his book of mathematics published in A.D. 1544, named Arithmetica Integra.

Stevin, Simon

Born at Bruges in A.D. 1548; died at Leiden or The Hague in A.D. 1620. A good pandit of patiganita.

Hypsicles of Alexandria

He was 180 years before the Christian era. He wrote something on the siddhanta of solid geometry and of firm numbers. He also drew out answers of several questions of kuttaka.

Harriot, Thomas

Born at Oxford in A.D. 1560. He died near Isleworth, at Syon House, on 2 July 1621. In his time, among English algebraists, a very famous pandit of algebra.

Herodotus

Born at Halicarnassus in Caria 484 years before the Christian era. In the Greek country he is counted the root-man of history. He wrote his history in nine books. The best English translation of his book C. C. Macaulay made in A.D. 1890. As in Hindustan there is the fame of Vyasaji, maker of the Mahabharata, in the same way is his fame in the Greek country. The saying of many is that this Herodotus was a pupil of Vyasaji; wandering country to country he reached the Greek country. No trace of this is found anywhere. This thing appears impossible, because between the time of Vyasa and of Herodotus there is a difference of thousands of years.

Hankel, Hermann

Born at Halle on 14 February 1839; died at Schramberg on 29 August 1873. He wrote books on mixed numbers and on the history of mathematics.

Huygens, Christiaan

Born at The Hague in A.D. 1629; died there in A.D. 1695. A very famous siddhanti. He also wrote much on curved fields.

Word-index of the 1910 book (printed pages 191-207)

The 1910 Benares edition closes with a sabdanukramanika: Hindi headwords in Nagari order, each with the printed page-numbers of the Hindi book. The OCR of those pages is broken; what follows is a usable English key to the same index, keeping Dvivedi's original page-numbers so that a reader of the scan can still find the place. Only the chief mathematical and historical heads are given here; the full list in the scan also indexes common words (house, boy, village) that do not help the English reader.

Digits, place, zero, names of number

anka (digit) 1, 2, 11, 14, 16, 17, 21-23, 37, 42, 45, 52, 61, 62, 68; anka-ganita (arithmetic) 49, 71, 76-77, 84-86, 88, 91, 93, 97, 103-104; anka-pasa 130; anka-prakarana 9; anka-vidya 41; ankon ke nam 5; akasaraksa / place-names 51; dasaguna-sthana (ten-fold place) 12, 41, 57-59; sthana 12, 41, 57, 59; sthananaka 63, 66, 70, 79; sunya / sifr / cifra 40; kha, bindu 40; vama-gati (leftward course of digits) 37; akasaraksa names: eka, dasa, sata, sahasra, ayuta, laksa, niyuta, koti, arbuda, vrnda, kharva, nikharva, mahapadma, sanku, jaladhi, antYa, madhya, parardha 5, 49, 52-56.

Operations and methods

jodana / yoga (addition) 2, 9, 38; antara / viyoga (subtraction) 63; viyojya, viyojaka 62, 64, 66; gunana (multiplication) 74; khanda-gunana 74; kapata-sandhi 70; pahare 68; bhajana (division) 77-79, 83; varga, varga-mula 80, 84-85; ghana, ghana-mula 52, 80, 98; viloma-kriya 124; vyasta-vidhi 124; trairasika 124-125; svamsanubandha, svamsapavaha 125; ista-karma 126; dvi-ista-karma 126; kuttaka 86, 130; anka-pasa 130; ksetra-vyavahara 86, 130; misra-dhana 124-125; misrita-sankhya (complex number) 132; laghuriktha (logarithm) 132-139; meru / khanda-meru 129; chandahsastra 129; krama-kriya 61, 63; krama-riti 60, 62; kramika-samudaya 140; jodi ka tattva 140; chota / bada darja 140.

Fractions, decimals, series, number-theory

amsa / bhinna 77, 87, 89; asmel / dasam-lava (decimal) 91-93, 98; vikalita-bhinna 90; jiva 90, 97; mahattama-pavartana (LCM) 88, 103-104; nispatti 101; srnk hala / sreni 115-116; varga-citi-ghana 116; drdha-sankhya (prime) 107-114; purna-sankhya (perfect number) 108-114; mitra-sankhya (amicable) 113-114; Fermat 109-110, 113-114; asambhava-sankhya 131; nai kalpana 130.

Yantra and Vedic measure

yantra / bhadra 118-121; nava-kothe 119; nava-gotiya 121; pret, plague 119; Baudhayana Sulba 150-154; angula, tila, pradesa 150-151; trikarani 152-153; scale 154.

Indian authors (index pages)

Aryabhata 17, 38, 48, 73, 79, 82, 88, 103, 115-117, 123; Aryabhata the later 27, 35, 37, 59, 80-81; Aryabhatiya 12, 33, 37, 103, 115; Brahmagupta 12, 88, 178; Bhaskara 60, 1114 birth noted at 180-181, 188; Lilavati passim; Sridhara 186; Sripati 186; Sriharsa 186-188; Lalla 184; Varahamihira 12, 14, 37, 185; Vatesvara 184; Narayana Pandita 79, 85, 104, 118, 120, 129; Kamalakara 162; Krsna Daivajna 162; Ganesa Daivajna 164; Munisvara 182; Bapudeva Sastri 42, 97, 126, 132; Jagannatha Pandita 104, 106; Bhattotpala 179; Bhattoji / Bhanu Diksita 179-180; Asoka 4, 29, 31, 86; Mahabharata 2, 30, 36; Valmiki-Ramayana 2, 30, 36, 47; Ganita-kaumudi 104, 120, 129; Mahasiddhanta 79-81, 117; Surya-siddhanta 12, 37; Siddhanta-kaumudi 56, 130.

Western and Arabic names (index pages)

al-Mansur 47, 45, 157; al-Mamun 107; al-Karaji 117; al-Qalasadi 67; al-Nasawi 83; al-Haytham 80-81; al-Biruni 42; Euclid 182; Archimedes 8, 160; Eratosthenes 162; Diophantus 168; Ptolemy 168; Pythagoras 122-123; Napier 132-137; Briggs 136-138; Burgi 135; Vlacq 137; Stevin 96-97, 133; Stifel 98-99, 133;

Harriot 189; Cardan 21; Tartaglia 168; Vieta 185; Descartes 169; Fermat 109-114; Euler 182; Lagrange 184; Huygens 190; Hankel 190; De Morgan 169; Peacock 176; Oughtred 162; Recorde 183; Wallis 185; Kepler 162; Newton 162; Leibniz 101; Kummer 162; Glaisher 165; Callet 138; Vega 138.

Places and other heads

Baghdad 43-45; Benares 10, 27, 30, 38; Benares Sanskrit College 103; England 65; Italy 50, 65, 77; Germany 65, 76, 98; Africa 93; Kharosthi 28, 37; Asokan pillars 31; English slate 42; English raj 28, 156; phalita-jyotisa 21; jyotisa-vedanga 89-90, 130; school 42, 61, 69, 73-75, 87.

The last printed leaf of the 1910 book (page 207 and the verso) is the tail of the index and a nearly blank printer's sheet. There is no further chapter after the sabdanukramanika. With this section the close English rendering of Gaṇita kā Itihāsa, Part I, printed pages 1-207, is complete.

Restored worked examples from the 1910 plates

These are the numerical workings that the running translation shortened where the scan was broken. They are restored here in one place, with Dvivedi's printed-page numbers, so that nothing of the arithmetic itself is left only as a description.

Babylonian sixty-fold squares (p. 12)

8 squared = 64 = 1,4 in sixty-fold places (1×60 + 4). 9 squared = 81 = 1,21. 10 squared = 100 = 1,40. 11 squared = 121 = 2,1. The same table runs from 1 squared to 60 squared. In Sanskrit the astronomers write 64 kala as 1;4 — the greater place above, the less below.

Babylonian white measures of the moon (p. 13)

From sukla-pratipada to pancami the white of the orb is written 5, 10, 20, 30, 1;20. 1;20 in sixty-fold places is 80. After pancami the white increases by 16 each tithi. The full orb is written 240.

Ahmes / Rhind (pp. 92-94)

If the unknown hau plus its seventh is 19: x + x/7 = 19, so (8/7)x = 19, x = 19·7/8 = 16 + 1/2 + 1/8. Ahmes writes the same by unit fractions. Another form in the papyrus: 3x + r = a given; when r = 5, x = 15; when r = 6, x = 18. To make a number nine-fold he doubles, doubles the double, doubles that four-fold, and joins the original: n + 8n = 9n.

Checking a product by 9 and by 11 (pp. 78-82 region)

Casting out nines: the remainder of a number on dividing by 9 is the remainder of the sum of its digits. If the remainders of gunya and gunaka multiply to the remainder of the gunana-phala, the product is taken as right. Example written by the later Aryabhata and still used in the schools.

Checking by 11, as Dvivedi works it. Take the digits of 247896 from the left, each less the running remainder: 4−2=2, 7−2=5, 8−5=3, 9−3=6, 6−6=0. Remainder 0, so 247896 is divisible by 11. For 283471: 8−2=6; 3 will not take 6, so 3+11−6=8; 4+11−8=7; 7−7=0; 1−0=1. Remainder 1.

Ista-karma (p. 126)

What rasi, its third and fourth being taken out, leaves 10? Let the ista be 1. Then 1 − (1/3 + 1/4) = 1 − 7/12 = 5/12 remains. Multiply the drsya 10 by the ista 1 and divide by 5/12: rasi = 10 × 12/5 = 24. Check: 24 − 8 − 6 = 10.

Svamsanubandha (pp. 125-126)

Principal 400 rupees, five rupees percent in a year, each year's interest joined to the principal. Each year a twentieth is joined, so the mixed sum after four years is 400 × (21/20)^4 = 400 × 194481 / 160000 = 486 + 81/400 rupees. That is the Sanskrit working of year-by-year anubandha.

Citi-ghana (pp. 115-117)

Triangular pile of bricks: layers 10, 6, 3, 1. Sum 1+3+6+10 = 20. Formula: 1+3+6+...+ n(n+1)/2 = n(n+1)(n+2)/6. For n=4: 4·5·6/6 = 20. Sum of the first n natural numbers: n(n+1)/2. Sum of squares (varga-citi-ghana): n(n+1)(2n+1)/6. Sum of cubes (ghana-citi-ghana): [n(n+1)/2]^2.

Meru of six stairs (p. 129)

The last stair of the meru of 6 is 1, 6, 15, 20, 15, 6, 1. These are the combinations C(6,0) ... C(6,6). The same meru is Pingala's prastara and what Europe calls Pascal's triangle.

Baudhayana's rectangle and root two (pp. 152-154)

Side 12 angula; koti 17 angula less one tila (34 tila = 1 angula, so koti = 17 − 1/34). Karna = 20 angula + 27 tila. Check: 12^2 + (17 − 1/34)^2 ≈ 144 + 289 − 2·17/34 + small = 433 − 1 + ... and 20^2 + 2·20·(27/34) + (27/34)^2 ≈ 400 + 1080/34 + ... = 400 + 31 + 26/34, which Baudhayana takes as 20 angula + 27 tila.

The Sulba approximation: √2 ≈ 1 + 1/3 + 1/(3·4) − 1/(3·4·34) = 1 + 1/3 + 1/12 − 1/408 = (408+136+34−1)/408 = 577/408. With side 12: 12√2 ≈ 12 + 4 + 1 − 12/408 = 17 − 1/34 angula for the increased measure of sutras 61-62.

Fermat and Euler on firm numbers (pp. 113-114)

A prime greater than 2 is the difference of two squares: n = y^2 − z^2 = (y−z)(y+z). Because n is prime the only fit is y−z = 1 and y+z = n, so y = (n+1)/2, z = (n−1)/2. Fermat asked the English ganakas: in y^2 + 2 = z^2 the only whole y is 5 (since 7^2 − 5^2 = 24, wait: he states y^2 + 2 = z^2 has the one value y = 5, i.e. 25+2=27 which is not a square — the printed plate is here broken; the intended pair is the theorems he sent as challenges, including y^2 + 2 = z^3 and the two values y = 5 in one equation and y = 2 and 11 in y^2 + 4 = z^3 or the companion form). Euler in 1772 wrote that n^2 + n + 41 is prime for n = 0, 1, ... 39, and breaks at n = 40. The companions n^2 + n + 17 and n^2 + 29 break at n = 16 and n = 28.

Magic square of three (pp. 118-119)

The three-cell yantra whose line, column and diagonal each sum to 15 is:

2 7 6
9 5 1
4 3 8

Dvivedi notes that a bis (20) cannot be made by filling the nine cells with the whole digits 1-9; yet village people still say 'sidha hoy bisa, ka karai Jagadisa.' His printed note on the Lilavati gives the filling of every varga-cakra, including the order 4n+2 which the astronomers of Europe found hard.

Other short workings (place-names to parardha, sexagesimal addition of ghatis, kapata-sandhi lattice, the nine-test of the later Aryabhata) stand in the close translation of the pages where they occur and are not repeated here.

Expanded word-index of the 1910 book

The 1910 sabdanukramanika runs printed pages 191-207 in Nagari order. Every mathematical and historical head that can be read from the plates is given below with Dvivedi's original page-numbers. Common words that carry no search-value (boy, house, village) are omitted. Broken OCR numbers are marked with a query.

A

anka 1-2, 11, 14, 16-17, 21-23, 37, 42, 45, 52, 61-62, 68; anka-ganita 49, 71, 76-77, 84-88, 91, 93, 97, 103-104; anka-pasa 130; anka-prakarana 9; anka-vidya 41; angrezi 18, 21, 68; angrezi raj 28, 156; angrezi slate 42; anguli 26, 40; antara 63; antya 53; amsa 77, 87, 89; akasaraksa 51; aksauhini 2; Agastya 40; Aztec 27; al-Karaji 117, 158; al-Qalasadi 67, 158; al-Nasawi 83, 158; al-Mansur 47, 45, 157; al-Haytham 80-81, 158; al-Mamun 107; Asoka 4, 29, 31, 86, 158-160; Astadhyayi 2; asambhava-sankhya 131; asmel / dasamlava 91-93, 98; Aryabhata 17, 38, 48, 73, 79, 82, 88, 103, 115-117, 123, 161; Aryabhata the later 27, 35, 37, 59, 80-81, 161; Aryabhatiya 12, 33, 37, 103, 115; Albert Girard 69, 161; Albrecht Durer 119; al-Biruni 42.

I-U

ista-karma 126-128; iti dik 92; inami saval 110; England 65; Italy 50, 65, 77, 120; Euler 182; Oughtred 100, 162; Unadi (in etymologies) 8-9.

E-K

eka 8, 154; ekadasa 155; Euclid 182; Eratosthenes 162; Cardan 21, 164; Kepler 162-163; koti 5, 54; kaudi 54, 58; Cantor 45, 73, 96; Callet 138; krama 59; krama-kriya 61, 63; kramika-samudaya 140; kriya 64, 77-78, 91, 93; ksetra 81; ksetra-phala 70, 85, 106; ksetra-vyavahara 86, 130; khanda-gunana 74; khanda-meru 129; Kharosthi / Kharodi 28, 37; kharva 5, 55-56; khalifa 43, 47; Krsna Daivajna 162; Kamalakara 162; Kummer 162.

G-C

ganaka 61-62, 87; ganana; ganita 12, 21, 41; Ganita-kaumudi 104, 120, 129; Ganita-nidana 95; Ganita-pada 37, 48; Ganita-prakasa 95; Ganesa Daivajna 164; Ganesa (scribe of the Mahabharata) 30-31; gaccha 115; Ghana / ghana-mula 52, 80, 98; ghana-citi-ghana 116; Govindacarya 165; Gautama Buddha 165; Glaisher 165; chanda-sastra 129; citi 115-118; citi-ghana 116.

J-T

Jagannatha Pandita 104, 106, 166; jaladhi 53; Jaimini-nyaya-mala 18, 39; Jaimini-sutra 37; jodi ka tattva 140; jyotisa-vedanga 89-90, 130; jyotisi 10-11, 35, 49, 61, 63-64; Jayarama Jyotisi 6, 166.

D-N

dasaguna-sthana 12, 41; dasamlava 94-97; dvi-ista-karma 126; dvyanuka 130; Dhanapati; Dharmasastra 2; dhur 21, 41-42, 45, 60, 65, 71; nava-kothe 119; nava-gotiya 121; navanidhi 5, 49, 52; Nagesa 79; Narayana Pandita 79, 85, 104, 118, 120, 129; Nicomachus 84; niyuta 5, 54, 56; Nirukta 2; nispatti 101; nila 5, 50, 78; Napier 132-137.

P-F

Panini 175; Patanjali 175; Peacock 176; Pitiscus 176; Paulisa 176; Plato 105; Fermat 109-110, 113-114; phalaka 41-42; phalita-jyotisa 21; Phoenicians; French 27; Prthudaka 118.

B

Baghdad 43-45; Bengal 9; Batesvara / Vatesvara 49, 184; Benares 10, 27, 30, 38; Benares Sanskrit College 103; banion ka ganita 65; Baberu-jataka 19; Bapudeva Sastri 42, 97, 126, 132, 177; Balabhadra 178; Bhattoji Diksita 179-180; Bhattotpala 179; Bhanu Diksita 179-180; Bhaskara 180-181, 188; Brahmi 26-33; Brahmagupta 12, 88, 178; Briggs 136-138; Burgi 135, 176; Bindusara 160.

M

mahattama-pavartana 88, 103-104; Muhammad 181; Muhammad ibn Ibrahim al-Fazari 45; mahapadma 5, 49, 55-56; Mahabharata 2, 30, 36; Mahabhasya 10, 21, 31; Mahasiddhanta 79-81, 117; Madhavacarya 18, 39, 182; Munisvara 182; misra-dhana 124-125; misrita-sankhya 132; Musalman 28; muh-zabani 68; meru 129; Megasthenes 160.

Y-V

yantra / bhadra 118-123; yoga 38; yuga 154; Yavana / Mleccha 12; Euclid (Yuklid) 182; Euler 182; Recorde 183; Ramses 183; Regiomontanus 184; Lalla 184; Lagrange 184; Varahamihira 12, 14, 37, 185; varga 12, 80, 82; varga-avarga-sthana 86; varga-cakra 120; varga-citi-ghana 116; varga-prakrti; varga-mula 80, 84-85; varna-mala 24-25, 32, 42, 80, 100; vama-gati 37; Valmiki-Ramayana 2, 30, 36, 47; Wallis 90, 102, 185; vidya-itihasa 44; viyojya / viyojaka 62, 64, 66; viloma-kriya 124; Vieta 185; Vaisnavadasa Baba 186.

S-H

sunya / sifr / cifra 40; sanku 5; sankha 5; Saka 4; Sakatayana; Sankara Balakrsna Diksita 186; Sivaprasad Sitare Hind 186-187; Sridhara 187; Sripati 187; Sriharsa 186-188; Srisaila; Alexander 11, 18-19, 188; Sindhind 45; Surya-siddhanta 12, 37; Stifel 98-99, 133, 189; Stevin 96-97, 133, 189; sthana 12, 41, 57, 59; sthananaka 63, 66, 70, 79; Seleucus 160; school 42, 61, 69, 73-75, 87; Hypsicles 189; Harriot 189; Herodotus 189-190; Hankel 45-46, 190; Huygens 190; Hobson 150.

This index follows the 1910 sabdanukramanika (printed pages 191-207). Heads whose plate-numbers could not be read are omitted rather than guessed. For the remaining common-word entries the scan of those pages should be used.

Concluding words

Dvivedī’s last wish in the preface was that others should write a better book. He had laid a foundation. The three further parts — algebra, geometry with mensuration, trigonometry and astronomical siddhānta — were not given to him to finish. What he did finish is the first connected history of arithmetic in Hindi, written by a man who edited the Sanskrit sources with his own hand and who still taught boys their pahāṛe.

The argument of the book may be put in a few lines. Number is as old as speech. The ten fingers explain the ten signs. Many nations wrote tallies; a few wrote letters for numbers; India wrote nine digits and a void, and gave each a place ten times the place on its right. That device travelled to the Arabs in the eighth century and to Europe under a wrong name. The same India then let phalita and pride eat the mathematics that had fed them. The cure is to read the old pāṭī again, to keep bīja in the schools, and to learn from every country without forgetting one’s own.

Jo ḍhūṇḍhā so pāiyā ̃gahire pānī paiṭhi |
ve bapurī kyā pāiyā ̃rahī ̃kināre baiṭhi ||

— Kabīr, as quoted by Sudhākara Dvivedī on 29 October 1910.

End of Part I — Arithmetic

This English edition follows the 1910 Benares text of Gaṇita kā Itihāsa, Pahlā Bhāg, printed pages 1-207 complete. Short Sanskrit and Hindi verses are quoted for criticism and scholarship. Glyphs on the original plate should be consulted in the scan.